Fuzzy Color Spaces: A Conceptual Approach to Color Vision

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***Abstract*—In this paper, we introduce formal definitions of the concepts of fuzzy color and fuzzy color space. First, we formalize the notion of fuzzy color for representing the correspondence between computational representation of colors and perceptual color categories identified by a color name. Second, we propose a methodology for learning fuzzy colors based on the paradigm of conceptual spaces, where prototypes are used for each category to be learnt. Since the conceptual space approach yields crisp categorizations, we introduce a novel methodology for defining fuzzy boundaries of color categories on the basis of a Voronoi tessellation of a color space. Finally, we also formalize the notion of fuzzy color space as the collection of fuzzy colors corresponding to the color categories employed in a certain context/application and/or for a specific user. Different typologies of fuzzy color spaces are proposed in order to be consistent with the nature of the categories we want to model. Our approach is illustrated by defining fuzzy color spaces using RGB with the Euclidean distance. Examples based on the well-known ISCC-NBS color naming system are presented, as well as others based on collections of color names and prototypes provided by users. The proposal is evaluated and compared with the most used approaches for color modeling. Additionally, a website located at http://www.jfcssoftware.com including all experimentation data, software implementing our models, and additional materials is available to researchers in color modeling.**

***Index Terms*—Conceptual spaces, fuzzy color categorization, fuzzy colors, fuzzy color space, fuzzy image processing.**

I. Introduction

COLOR vision is the branch of cognitive science devoted to the study of color perception, involving multiple disciplines (physics, psychology, physiology, computer science, linguistics, genetics, and anthropology) [1]–[3]. Computer science, more specifically cognitive computing and within it computer vision, is concerned with different aspects of color vision. Information processing ideas coming from computer science and information theory are employed as a paradigm for theorizing about the nature of the human mind as a computational process [3]. Within this paradigm, which has been applied to visual perception as well as to other cognitive phenomena, computers have had two main roles: as the primary theoretical analogy for mental processes, and as the preferred tool for testing new theories of color vision [3], by developing computational systems able to imitate humans in certain tasks involving color perception. The objectives of such systems are processing color information in the same way as humans do, and contributing to a more natural computer–human communication.

Following [4], at the computational level, color perception has as input the light reflected into the eye by surfaces in the environment, as output color experiences that arise when an observer views those surfaces, and the mapping is the psychophysical correspondence between the two [3]. Computer science introduces a third element in the game: the light is captured by cameras and transformed into digital images, in which color of pixels is represented by vectors of numbers in some suitable color space.

A system able to imitate a human should be able to, given a region in a captured image, generate the same expression representing the color perception as that the human would do by looking to the corresponding surface in the real world. Such an expression is a color name or color term [5]–[9]. One problem that appears in such systems is related to the color constancy, a phenomenon of our brains that allows us to perceive a certain surface as homogeneous in color despite the amount of light reflecting from different parts of the region. Computers distinguish differences between pixels in the area corresponding to the surface. Hence, the perceived color in a given region depends on the interaction between colors of pixels and their distribution in the region. Similar problems are derived from other perceptual illusions generated by the human perceptual system.

Let us assume that the color information regarding a certain surface or image region is mapped to a single color as a vector in a color space, eliminating the effect of illumination and other perceptual problems. The problem we face in this paper is: How can we generate automatically a correct expression of the color perception using linguistic color terms? That is, how can we find a suitable mapping between digital representation of colors, and color terms employed by humans? For example, in Fig. 1, the circled colors are represented in computers by the

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Fig. 1. Image of a gradient between two colors.

triplets $[248, 29, 82]$ and $[128, 68, 245]$ (in RGB) while users employ the linguistic terms "red" and "purple" to describe them, respectively. This problem is an example of what is known as the semantic gap [10], a problem involving issues like modeling color properties by using machine learning techniques, defining color similarity, etc. Solving this problem is very important in applications like human–machine interaction (for instance, generating linguistic descriptions of images in natural language [11], [12]), image segmentation (for example, determining "red" regions in an image), visual information retrieval [13], and a long etc.; see [14].

Color naming, which is located between the visual perception and language disciplines [15], is an approach for solving the above-mentioned semantic gap [16]. The objective is to define a correspondence between the domain of computational representations of color and a multidimensional space of linguistic labels corresponding to color names. Color naming can, in principle, be solved by performing a quantization of a color space, that is, performing a partition of the color space in terms of similarity and assigning a color name to each group of vectors in the partition. The semantics of the color term is represented by the corresponding group of vectors.

Unfortunately, the ordinary approach of using a single crisp partition of the color space is not a solution for the color naming problem, since it is not able to reflect the cognitive process of color categorization performed by humans, which has the following characteristics.

1) Color categories have ill-defined boundaries, that is, they do not correspond to crisp sets of vectors. In fact, color categories are known since long ago to be paradigmatic examples of fuzzy concepts that arise from our mental processes [17]–[19]. For instance, in Fig. 1, it is not immediate to establish a precise boundary separating the crisp colors corresponding to the color terms "red" and "purple," but the boundary is fuzzy. Further, there are colors, especially in the central area of the image, for which a human could not tell which of the two color names (associated with extreme colors) is the right one. Possibly, the user will say that they are both "red" and "purple" to a certain degree. Fuzzy set theory has been shown to be a suitable approach for representing such concepts in [20], refuting previous works that held the opposite [21]. Hence, a quantization of a color space using crisp sets of vectors is not a suitable solution for color categorization. As we shall see in Section II, different approaches to the fuzzy categorization of colors have been employed in the literature.
2) The color categories that we are able to distinguish are subjective. Colors perceived are psychological properties of our visual experiences when we look at objects and lights that we are able to name [3]. Hence, from a philosophical point of view, colors are relational because they can be specified only in relation to the visual experiences of a perceiver. That is, color categories are different for each person, and finding the appropriate computational representation of such color categories for a person requires its participation. Hence, using experiments with users in order to obtain suitable representations of color categories is very common in the literature, as we detail in Section II.
3) Color categories are context dependent. For instance, the color category corresponding to the color term "red" is different for the contexts of wine colors, hair colors, star colors, and fruit colors, even for the same person. Following the illustrative example in Fig. 1, if the circled colors have to be named, the answer may be different depending on the subject and context (e.g., a winemaker could name them as "vermilion" and "purple," while a greengrocer could use terms as "strawberry" and "aubergine"). Not only the particular definition of each color category, but the set of color categories we use is different in each context (the set of colors employed for wines is different from that employed for hair, etc.). A particular case of context is a cultural environment, in which we can have a collection of color names for general purpose that is agreed, with very similar representations, by a group of people; many of the proposals in color naming are devoted to defining the color categories for a general cultural environment.

In this paper, we introduce a new approach to solve the semantic gap problem in the perception of color, a significant extension of our proposal in [22] with important innovations with respect to existing approaches. First, we formalize the notion of fuzzy color as the basic mathematical object for representing the correspondence between computational representation of colors as vectors in a color space, and perceptual color categories identified by a color name. From now on, we shall refer to vectors in a color space as crisp colors. Second, we introduce a methodology for learning fuzzy colors based on the paradigm of conceptual spaces [23], [24], based on using prototypes for each color category to learn. Since the conceptual space approach yields crisp categorizations, we introduce a novel approach for obtaining fuzzy boundaries for fuzzy colors. This approach is in accordance with the principle stated in [25]: "Categorization is based on the generally accepted notion that [fuzzy] colors have prototypes and a region surrounding each prototype [18] with fuzzy boundaries [19]," but using a different approach to the adaptive networks employed in [25]. Our methodology avoids important problems of current methods: We do not rely on membership functions having geometrically regular forms [13], [26]–[29], which are not always in accordance with the correspondence between crisp colors and color names; also, we do not require tedious experiments and/or a large number of observers [30]–[33].

As a third innovation, prototypes are provided by humans, and hence, our methodology reflects the subjectivity in the perception of color. However, an advantage with respect to existing approaches is that the participation of humans is not achieved through tedious and complex experiments, but by providing a few pairs (prototype, color name). In addition, we introduce the use of positive and negative prototypes for a color category, allowing each fuzzy color to be learnt independently from other colors.

Finally, we also formalize the notion of fuzzy color space as the collection of fuzzy colors corresponding to the color categories employed in a certain context/application and/or for a specific user. Note that a fuzzy color space is not a fuzzy vector space, but a fuzzy conceptual space in the sense of [23] and [24]. Different typologies of fuzzy color spaces (that we call partition, disjoint and covering), their properties, and how to learn them are issues we discuss in this paper. Fuzzy color spaces go beyond existing proposals, which focus on a limited set of color terms for a widely accepted cultural environment, mainly the 11 basic color terms determined by Berlin and Kay [34] that define a fuzzy partition of the space of crisp colors. Typologies of fuzzy color spaces not corresponding to fuzzy partitions are described here, with practical situations for which they are better suited, which are needed to be consistent with the different nature of the colors terms we want to use. Such fuzzy color spaces include color categories with nonexclusive semantics (for instance, those corresponding to color names lemon and yellow). In addition to defining a categorization, a fuzzy color space also induces a similarity relation between crisp colors.

The rest of this paper is organized as follows. In Section II, related works about color naming using fuzzy sets are analyzed. In Section III, formal definitions of the concepts of fuzzy color and fuzzy color space are introduced. Section IV details our methodology to learn fuzzy color spaces. Some examples of construction of different typologies of fuzzy color spaces are illustrated in Section V, and, finally, the main conclusions are summarized in Section VI.

II. Related Work

In general, in the literature, we can distinguish three main types of computational approaches to model color terms, depending on the type of association between color stimulus (represented by a vector using a color space) and color terms (represented as linguistic labels). These are models based on a crisp quantization of the space [35]–[37], probabilistic models that calculate the probability that a stimulus is assigned to a color term [38]–[41], and fuzzy models that assign a membership degree to a color term.

In this paper, we focus on fuzzy models where a color term is modeled by a fuzzy set. Since Kay and McDaniels [19], who were the first to propose a color naming model using a fuzzy approach, many authors have proposed fuzzy approaches for modeling the color, which are distinguished mainly in the way the membership functions are defined. These are, on the one hand, models where the parameters of the membership functions are adjusted using perceptual experiments and, on the other hand, those that define the membership functions without experiments, usually by means of equidistributed trapezoidal functions or combinations of them over the color components.

The models based on perceptual experiments need large amounts of data from psychophysical experiments to learn the parameters. These experiments require a large number of subjects and color stimuli and devices under a properly calibrated standard lighting conditions. For instance, Seaborn *et al.* [31] use the fuzzy c-means algorithm to model color terms, using the data from the psychophysical experiment of Sturges and Whitefield [42], where 20 subjects with a total number of 17 840 observations of 446 colors were involved in the task of identifying colors in the Munsell Space. Yendrikhovskij [43] also uses the fuzzy c-means algorithm to model color terms, where the number of clusters is the number of colors provided by the users, and the membership degrees are assigned based on the distance to the centroid of each cluster. Benavente [32] uses a parametric model, where each color term is modeled as a fuzzy set defined by sigmoid functions, whose parameters are estimated by an adjustment process using data from several psychophysical experiments.

On the other hand, the membership functions in the models that are not based on perceptual experiments are usually defined by 1-D trapezoidal functions for each color component. For example, several authors define trapezoidal functions (generally equidistributed) on the components of the color space HSV or HSL, specifically over saturation, brightness, and hue components, or combinations of them [13], [26]–[29], [44], [45]. Kim and Lee [46] propose a fuzzy color model, where the core of a fuzzy color is a sphere and the membership degree is defined as a linear function on the basis of the Euclidean distance to the center of the sphere.

In general, approaches based on perceptual experiments provide good models of color terms. However, the color modeling requires, and is conditioned to, tedious psychophysical experiments, which limit the representativeness of the models. As a consequence, usually, only the 11 basic color terms are modeled. On the other hand, approaches that do not require experiments to learn the parameters of the membership functions impose regular forms on these functions, when it is known that the representation of many color terms has an irregular form [47], [48]. Hence, in most of the cases, these models do not correspond with human intuition and do not model correctly the subjective nature of color.

III. Formalization

In this section, we introduce formal definitions of the notions of fuzzy color and fuzzy color space, the different typologies of spaces, and their properties.

A. Fuzzy Color

Let us consider the set of crisp colors that can be represented in a computer using any crisp color space. A fuzzy color is the computational representation of a color term defined and named by humans. Formally, we have the following.

Definition 3.1: A fuzzy color $\widetilde{C}$ is a linguistic label whose semantics is represented as a normal fuzzy subset of colors.

Imposing the use of normal membership functions implies that for each fuzzy color $\widetilde{C}$, there is at least one crisp color

$\mathbf{r}$ such that $\widetilde{C}(\mathbf{r}) = 1$. As a consequence, no fuzzy color is represented by the empty set. We require normalization since we expect that at least one color is fully representative of a color term.

B. Fuzzy Color Space

A fuzzy color space is a collection of fuzzy colors represented by membership functions in a conceptual space [23]. Formally, we have the following.

Definition 3.2: A fuzzy color space is a crisp set of fuzzy colors.

In practical applications, it is usual to work with different color terms, whose number and design depend on the application itself. In this sense, the concept of fuzzy color space is useful, among other things, for representing the set of fuzzy colors that are relevant to a certain application.

Let $\widetilde{\Gamma} = \{\widetilde{C}_1, \ldots, \widetilde{C}_m\}$ be a fuzzy color space with fuzzy colors defined on a certain crisp color space Γ. We introduce the following definitions.

Definition 3.3: $\widetilde{\Gamma}$ is a covering space iff for some t-conorm

$$\bigcup_{\widetilde{C}_i \in \widetilde{\Gamma}} \widetilde{C}_i = \Gamma. \tag{1}$$

It is immediate that covering spaces satisfy $\forall \mathbf{c} \in \Gamma\ \exists \widetilde{C}_i \in \widetilde{\Gamma}$ such that $\widetilde{C}_i(\mathbf{c}) > 0$.

Definition 3.4: $\widetilde{\Gamma}$ is a disjoint space iff $\forall \widetilde{C}_i \in \widetilde{\Gamma}$, $\forall \mathbf{c} \in \Gamma$, $\widetilde{C}_i(\mathbf{c}) = 1$ implies $\widetilde{C}_j(\mathbf{c}) = 0\ \forall i \neq j$.

Since fuzzy colors are normal fuzzy sets by definition, disjoint spaces satisfy $\widetilde{C}_i \not\subseteq \widetilde{C}_j\ \forall i \neq j$, since the cores of fuzzy colors have empty intersection. Notice that, although fuzzy colors in a disjoint space can have nonempty intersection, the intersection will not be a fuzzy color, since it will be a subnormal fuzzy set.

Definition 3.5: $\widetilde{\Gamma}$ is a partition space iff it is covering and disjoint.

Most of the works to represent color terms in the literature attempt to obtain partition spaces containing the basic color terms in the sense of Berlin and Kay [34], with small variations in the number of colors. In most of the cases, the idea is to find a representation of each basic color term that can be agreed on by most of people in a given context. However, our work is more general in that we want to provide a methodology for representing any color term, not only basic color terms, either given by an individual person or agreed by a collective in any particular cultural or application context. We also consider that it is not mandatory for fuzzy color spaces to be partition spaces, our methodology allowing for obtaining both partition and nonpartition spaces.

IV. Learning Fuzzy Color Spaces

Our approach yields fuzzy colors starting from simple information provided by the user and is based on the conceptual space paradigm [23]. This paradigm is suitable for representing and automatically learning color properties (see Section IV-A), but an extension to the fuzzy case is needed to obtain fuzzy colors and fuzzy color spaces (see Section IV-B).

A. Conceptual Spaces

The theory of conceptual spaces [23], [24] is a framework for representing concepts. A conceptual space is a metric space comprised by a set of objects described by n dimensions, each dimension representing a certain quality of an object, plus a metric. For instance, the color space RGB is an example of a conceptual space, where objects are colors described with three dimensions, each dimension representing the qualities of how much red, blue, and green a certain color has, plus Euclidean distance as a usual metric.

In the theory of conceptual spaces, two or more dimensions are said to be integral when one cannot assign a value to an object in one dimension without giving it values on the other dimensions. This is the case, for instance, with the dimensions in a crisp color space, since one cannot describe a crisp color by giving only the coordinate in one single dimension in general. A subspace of a metric space defined by a set of integral dimensions is called a domain. Crisp color spaces (RGB, HSI, CIE Lab, etc.) are examples of domains.

A particular type of concepts that can be represented in conceptual spaces are the so-called properties. A property is a concept represented by a region in a single domain. For instance, concepts representing color terms are color properties. In the following, we shall use color property and color concept interchangeably, referring to the representation of color terms in a conceptual space.

In [23], Gärdenfors argues for several advantages of the theory of conceptual spaces when modeling concepts. First, it allows us to overcome several difficulties of the symbolic approach and associationism/connectionism for representing concepts, quoting [23], "concept learning is closely tied to the notion of similarity, which has turned out to be problematic for the symbolic and associationistic approaches."[1]

Another advantage, which has motivated the use of the paradigm of conceptual spaces, is that conceptual spaces provide a paradigm to learn the representation of properties. The representation of a collection of properties can be obtained on the basis of the following.

1) An appropriate conceptual space, for example, suitable dimensions and a metric to represent the semantics of the properties.
2) A limited collection of points in the conceptual space as representatives (at least one) for each property, whose centroid define a prototype for the property.[2]
3) A Voronoi tessellation [49] of the metric space based on prototypes. A Voronoi tessellation consists of a partition of the space in regions named Voronoi cells, one for each prototype, such that a point in the conceptual space belongs to the property associated with the closest prototype determined in terms of the metric employed in the con-

[1]Gärdenfors proposes a view of concept representation with three connected levels, or perspectives, with different scale of resolution. In his view, representations using conceptual spaces are situated between the symbolic and associationistic representations.

[2]The learning approach for conceptual spaces is also based on ideas from the prototype theory of categorization developed by Rosch and collaborators [17], [18].

ceptual space. The boundaries of the Voronoi cells are points in space, which are equidistant from two (or more) prototypes.

For example, we can learn a representation for color terms using as conceptual space the RGB crisp color space plus the Euclidean distance, and as prototypes a collection of crisp colors for each color term we want to learn. In this case, the Voronoi tessellation consists of a partition of the RGB space into a collection of 3-D volumes (Voronoi cells), one for each prototype, each one containing those crisp colors that are closer to the corresponding prototype than to any other, according to the Euclidean distance.

However, the main problem with this approach is that, as pointed out in previous sections, Voronoi tessellations are crisp partitions with crisp boundaries. On the contrary, in many cases, the boundaries of the concepts to be learned are fuzzy [17]–[19], [23], as it happens with color terms. Hence, in order to obtain appropriate representations of color concepts (fuzzy colors as introduced in the previous section), crisp Voronoi tessellations alone are not enough.

The main idea we follow is that of "fuzzifying" Voronoi tessellations in terms of the metric defining the conceptual space so that the membership decreases with the distance to the prototype. In the next section, we introduce a methodology to obtain fuzzy Voronoi tessellations from classic tessellations as well as the calculation of the membership functions.

B. Methodology to Obtain Fuzzy Color Spaces

As we discussed in the previous section, a fuzzy color is a normal fuzzy subset of a color space, which can be defined by its membership function or equivalently, as it is well known, by all its α-cuts. Using this idea and the representation theorem of fuzzy sets, it is possible to define a fuzzy set by means of the collection of all its α-cuts. However, it is not always possible to define extensively α-cuts for all possible values $\alpha \in (0, 1]$. In this paper, we propose to define explicitly only some of these α-cuts (at least core) and the support and, then, to obtain the membership degrees of crisp colors by using an interpolation.

Let us consider a conceptual space, consisting of a crisp color space Γ plus a suitable metric d in Γ. In order to define a fuzzy color $\widetilde{C}$ representing a certain color term C, we require the following.

1) A crisp color $\mathbf{r}$, called the positive prototype, which is fully representative of the color term C, i.e., which membership to the fuzzy color must be one.
2) A set of volumes $\mathcal{V}_{\widetilde{C}} = \{V_1, \ldots, V_q\}$ with $q \geq 2$ and $V_i \subset V_{i+1}$ $\forall 1 \leq i \leq q-1$ corresponding to some α-cuts of $\widetilde{C}$ plus its support. In particular, let $\Omega_{\widetilde{C}} = \{\alpha_1, \ldots, \alpha_q\} \subset (0, 1]$ with $1 = \alpha_1 > \alpha_2 > \cdots > \alpha_q = 0$ be a set of levels, V_i corresponds to the α_i-cut of $\widetilde{C}$ for $1 \leq i < q$ (hence, V_1 is the core), while V_q is the support.
3) An interpolation mechanism for determining the membership function $\widetilde{C} : \Gamma \rightarrow [0, 1]$ on the basis of the positive prototype and the set of volumes that define the α-cuts and the support.

Once the above requirements have been established, the calculus of a fuzzy color space $\widetilde{\Gamma} = \{\widetilde{C}_1, \ldots, \widetilde{C}_m\}$, composed by a set of fuzzy colors, can be performed by obtaining each fuzzy color $\widetilde{C}_i$ individually.

In the following, the methodology to obtain a fuzzy color $\widetilde{C}_i$ whose positive prototype is $\mathbf{r}^i$ is detailed. Specifically, in Section IV-B1, we discuss about the selection of prototypes and the novel concept of negative prototypes. In our methodology, prototypes are used for calculating the volumes associated with $\widetilde{C}_i$. In Section IV-B2, we illustrate the calculus of the set of volumes $\mathcal{V}_{\widetilde{C}_i}$. In particular, we detail how to obtain the volume associated with the 0.5-cut (see Section IV-B2a), the volumes V_1^i and V_q^i corresponding to the core and the support (see Section IV-B2b), and the volumes for other α-cuts (see Section IV-B2c). In Section IV-B3, we propose the interpolation mechanism used for obtaining the final membership function. Furthermore, for a better understanding of our approach, the procedure is illustrated with examples in Section IV-B4. Finally, in Section IV-B5, we introduce some interesting properties for defining different typologies of fuzzy color spaces on the basis of different choices of the negative prototypes described in Section IV-B1.

1) Positive and Negative Prototypes: As we have mentioned before, our methodology is based on the theory of conceptual spaces [23], [24]; therefore, to learn a color term C_i, at least a representative color $\mathbf{r}^i$ associated with C_i (called positive prototype) and a Voronoi tessellation are required. To obtain that tessellation, prototypes of color terms distinct to C_i are also needed. In this sense, the tessellation is composed by one cell, which contains crisp colors belonging to C_i, while the rest of cells contain crisp colors that do not belong to C_i. However, while crisp colors associated with a color term C_i are similar to each other and to the representative $\mathbf{r}^i$ (positive prototype), colors that are not associated with C_i may be very different among them, i.e., the complement of a Voronoi cell is not usually another cell, but the union of several cells constituting a partition of colors, in terms of similarity, that do not belong to the color term.

As we shall see, restricting prototypes of cells that are not associated with a color term C_i to the positive prototypes of other color terms C_j with $i \neq j$, as it is usual in the paradigm of conceptual spaces, poses an important problem when the fuzzy color space to be obtained is not a partition space, i.e., when cores of fuzzy colors can overlap, or when there may be areas of the color space that do not correspond to any color term. In such cases, we need to consider prototypes that make up the complement of the color term C_i, which we intend to model, but that may not be used as positive prototypes of any other color term C_j. Therefore, in general, to obtain a fuzzy color $\widetilde{C}_i$, we will consider a positive prototype plus a set of representatives of colors that do not belong to the color term C_i (equivalently, they are in the term $\neg C_i$), which we call negative prototypes of the color term C_i. The latter can include positive prototypes of other terms that are disjoint to C_i.

Let R_i^+ and R_i^- be the set of positive and negative prototypes for learning the fuzzy color $\widetilde{C}_i$ corresponding to a color term C_i, respectively. In this paper, we consider $R_i^+ = \{\mathbf{r}^i\}$

composed by a unique positive prototype and a collection of negative prototypes $R_i^- = \{\mathbf{nr}_i^1, \ldots, \mathbf{nr}_i^k\}$ with $R_i^+ \cap R_i^- = \emptyset$.

2) Obtaining the Volumes Associated With $\widetilde{C}_i$: In order to obtain the set of volumes for a fuzzy color $\widetilde{C}_i$, we propose to calculate a Voronoi tessellation of the conceptual space Γ = RGB with Euclidean distance for the set of prototypes $R^i = R_i^+ \cup R_i^-$ (positive prototype and the set of negative prototypes). From the tessellation, we propose to obtain the volumes associated to some α-cuts of $\widetilde{C}_i$ and its support. In particular, in Section IV-B2a, we focus on obtaining the 0.5-cut. In Section IV-B2b, we propose how to obtain the core and the support, and in Section IV-B2c, we focus on obtaining any other α-cut.

a) Obtaining the 0.5-cut of the fuzzy color $\widetilde{C}_i$: In our approach, we propose to define the 0.5-cut of the fuzzy color $\widetilde{C}_i$ as the (crisp) Voronoi cell V^i corresponding to the positive prototype $\mathbf{r}^i$. This interpretation will provide fuzzy sets consistent with the natural criteria of assigning degrees greater than or equal to 0.5 to crisp colors that are closer to the prototype of the cell than to other prototypes (i.e., the crisp colors of the cell), and lower degrees to the rest (except for the crisp colors that are in the own boundary of the cells). Usually, the 0.5-cut will be one of the volumes employed in the definition of the fuzzy color, that is, $V^i = V_j^i \in \mathcal{V}_{\widetilde{C}}$ with $1 < j < q$.

The boundary of the Voronoi cell forms a convex polytope (in 2-D a polygon and in 3-D a polyhedron), satisfying that a crisp color in the boundary is equidistant from at least two crisp colors in R^i. An exception to this rule is the set of crisp colors on the planes, which limit the RGB cube because all Voronoi cells are truncated using the faces of the RGB cube. Note that the union of the rest of Voronoi cells (that is, the entire space minus the Voronoi cell associated with the positive prototype) defines the 0.5-cut of the fuzzy complement of $\widetilde{C}_i$.

The complexity of obtaining Voronoi tessellations in spaces of three dimensions ranges from $O(n^2)$ to $O(nlogn)$ in the number of prototypes, depending on the algorithm [50].

b) Obtaining the core and the support of the fuzzy color $\widetilde{C}_i$: We propose to obtain the volumes corresponding to the core and the support of $\widetilde{C}_i$ (V_1^i and V_q^i, respectively) by performing two scalings of V^i centered in $\mathbf{r}^i$, as follows.

Let P be a polyhedron. Let $\hat{P} = \Delta^{\mathbf{o},\mathbf{k}}(P)$ be the uniform scaling of P with respect to the point $\mathbf{o} = [o_1, o_2, o_3]$ and with scale factor $\mathbf{k} = [k_1, k_2, k_3]$, where for each point $\mathbf{v} = [v_1, v_2, v_3] \in P$, the corresponding scaled point $\hat{\mathbf{v}} = [\hat{v}_1, \hat{v}_2, \hat{v}_3] \in \hat{P}$ is calculated as $[\hat{v}_1, \hat{v}_2, \hat{v}_3, 1] =$

$$[v_1, v_2, v_3, 1] \cdot \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ (1-k_1)o_1 & (1-k_2)o_2 & (1-k_3)o_3 & 1 \end{bmatrix}. \tag{2}$$

On the basis of (2), the volume corresponding to the core is calculated by scaling V_i as $V_1^i = \Delta^{\mathbf{r}^i,\mathbf{k}_\lambda}(V^i)$, using as scale factor $\mathbf{k}_\lambda = [\lambda, \lambda, \lambda]$, with $\lambda \in [0, 1]$. On the other hand, the volume corresponding to the support is calculated as $V_q^i = \Delta^{\mathbf{r}^i,\mathbf{k}_{\lambda'}}(V^i)$, using as scale factor $\mathbf{k}_{\lambda'} = [\lambda', \lambda', \lambda']$ with $\lambda' \in [1, 2]$. In both cases, the scaling is performed with respect to the point $\mathbf{o} = \mathbf{r}^i = [r_1^i, r_2^i, r_3^i]$.

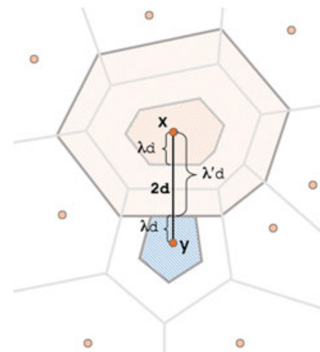


Fig. 2. Two-dimensional illustration of the role of the parameters λ and λ' in the definition of fuzzy models. In this example, $\lambda = 0.5$ and $\lambda' = 1.5$. In the figure, d represents the distance between $\mathbf{x}$ and the side of its Voronoi cell adjacent to $\mathbf{y}$ (consequently, the distance between $\mathbf{x}$ and $\mathbf{y}$ is $2d$). Note that the support associated with $\mathbf{x}$ and the core associated with $\mathbf{y}$ do not overlap because $\lambda + \lambda' = 2$. However, if $\lambda + \lambda' > 2$, they would overlap. The same happens to the rest of prototypes.

The condition $1 \leq \lambda + \lambda' \leq 2$ has been imposed for guaranteeing that the support of $\widetilde{C}$ will have empty intersection with the core of the negative regions when they are obtained using the same scaling value λ. In addition, $\lambda \in [0, 1]$ is imposed, since the core is included in the 0.5-cut, and $\lambda' \in [1, 2]$ since the 0.5-cut is included in the support. Fig. 2 shows graphically the influence of the parameters λ and λ', illustrating the case of $\lambda + \lambda' = 2$.

c) Obtaining any α-cut of $\widetilde{C}_i$: Previously, we have detailed the calculus of the volumes associated with 0.5-cut, core and support of the fuzzy color $\widetilde{C}_i$. Similarly, the volume V_j^i corresponding to any α-cut of $\widetilde{C}_i$ can be obtained by means of a scaling of V^i considering scaling factors with values between λ and λ'. Therefore, for any α-cut, the scaling parameter λ^α must satisfy that $\lambda^1 = \lambda$, $\lambda^{0.5} = 1$, and $\lambda \leq \lambda^\alpha \leq \lambda^\beta \leq \lambda' \; \forall \alpha > \beta$.

Volumes obtained by means of these scalings will form the set of polyhedra $\mathcal{V}_{\widetilde{C}_i} = \{V_1^i, \ldots, V_q^i\}$ with $q \geq 2$ and $V_j^i \subset V_{j+1}^i \; \forall 1 \leq j \leq q-1$ corresponding to certain α-cuts plus the support of the fuzzy set. As we discuss in the next section, the membership function is obtained by linear interpolation between the faces that limit the polytopes.

The number of the required polytopes depends on the form we consider most appropriate for the membership function. If $q = 2$, i.e., we only consider the polytopes that define the boundary of the core and the support, we have a linear membership function between the faces of both polytopes. As we shall see, this type of function is both simple and effective for fuzzy modeling. More complex functions, particularly nonlinear ones, can be approximated by including more polytopes.

3) Obtaining $\widetilde{C}_i$ by Means of Linear Interpolation: Let $\widetilde{C}_i$ with positive prototype $\mathbf{r}^i$ and $\Omega_{\widetilde{C}_i} = \{\alpha_1, \ldots, \alpha_q\} \subset (0, 1]$, with $q \geq 2$ and $1 = \alpha_1 > \alpha_2 > \cdots > \alpha_q = 0$ be a set of levels, and let $\mathcal{V}_{\widetilde{C}_i} = \{V_1^i, \ldots, V_q^i\}$ with $q \geq 2$ and $V_j \subset V_{j+1} \; \forall 1 \leq j \leq q-1$ be a set of volumes corresponding to the α_j-cuts of

$\widetilde{C}_i$ with $\alpha_j \in \Omega_{\widetilde{C}_i} \setminus \{\alpha_q\}$, plus its support defined by V_q^i. Let S_j^i $\forall 1 \leq j < q$ be the surface that limits the volume V_j^i of $\mathcal{V}_{\widetilde{C}_i}$.

The membership function is defined as follows:

$$\widetilde{C}_i(\mathbf{c}) = f(\mathbf{c}; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) \quad (3)$$

for each crisp color $\mathbf{c}$, where

$$f(\mathbf{r}^i; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) = 1 \quad (4)$$

and

$$f(\mathbf{c}; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) = \alpha_j \in \Omega_{\widetilde{C}_i} \quad \forall \mathbf{c} \in S_j^i \quad (5)$$

and for the rest of crisp colors, the membership degree is determined by linear interpolation in the segment of infinite length originating in $\mathbf{r}^i$ and passing through $\mathbf{c}$, we denote $\mathbf{r}^i\mathbf{c}+$, as follows: let $S_j^i \cap \mathbf{r}^i\mathbf{c}+ = \{\mathbf{s}_j\}$ be the intersection point between the surface S_j^i and the segment $\mathbf{r}^i\mathbf{c}+$. Let $d(\mathbf{a}, \mathbf{b})$ be the Euclidean distance between the crisp colors $\mathbf{a}$ and $\mathbf{b}$. Then, we have the following.

1) If $d(\mathbf{r}^i, \mathbf{c}) \leq d(\mathbf{r}^i, \mathbf{s}_1)$, then (the point is inside the core)

$$f(\mathbf{c}; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) = 1. \quad (6)$$

2) If $d(\mathbf{r}^i, \mathbf{c}) \geq d(\mathbf{r}^i, \mathbf{s}_q)$, then (the point is outside the support)

$$f(\mathbf{c}; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) = 0. \quad (7)$$

3) If $d(\mathbf{r}^i, \mathbf{s}_j) \leq d(\mathbf{r}^i, \mathbf{c}) \leq d(\mathbf{r}^i, \mathbf{s}_{j+1})$ with $1 \leq j \leq q - 1$, then (the point is between S_j and S_{j+1}), $f(\mathbf{c}; \mathbf{r}^i, \mathcal{V}_{\widetilde{C}_i}, \Omega_{\widetilde{C}_i}) =$

$$\alpha_{j+1} + (\alpha_j - \alpha_{j+1}) \left(\frac{d(\mathbf{r}^i, \mathbf{s}_{j+1}) - d(\mathbf{r}^i, \mathbf{c})}{d(\mathbf{r}^i, \mathbf{s}_{j+1}) - d(\mathbf{r}^i, \mathbf{s}_j)} \right). \quad (8)$$

Fig. 3 shows three examples in $\mathbb{R}^2$ of the membership function defined by (3)–(8) of a fuzzy model whose representative prototype is $\mathbf{r}^i$ in different directions. These examples correspond to segments $\mathbf{r}^i\mathbf{c}+$ for several points $\mathbf{c}$. In these examples, $\mathcal{V} = \{V_1, V_2, V_3\}$, corresponding to the core, the 0.5-cut and the support, have been considered. Thus, $\Omega = \{\alpha_1, \alpha_2, \alpha_3\}$, $\alpha_1 = 1, \alpha_2 = 0.5$, and $\alpha_3 = 0$, using $\lambda = 0.5$ and $\lambda' = 1.5$. Note that in these examples, due to the parameters λ and λ' used, the interpolation is linear between the surfaces that delimit the core and the support.

Note also that in the 3-D case, although the operations are based on 3-D vectors that represent crisp colors in RGB, and that polytopes are polyhedrons instead of polygons, the equations used would be the same in the 2-D case, since the interpolation is performed on a 1-D segment.

The procedure just described for defining a fuzzy color using RGB with Euclidean distance as conceptual space, with linear interpolation for the definition of the membership function, has been implemented efficiently and will be employed for illustrating our approach in the experimental part of this paper. Let us remark that the computation of lines, intersections, and distances is very efficient from the computational point of view.

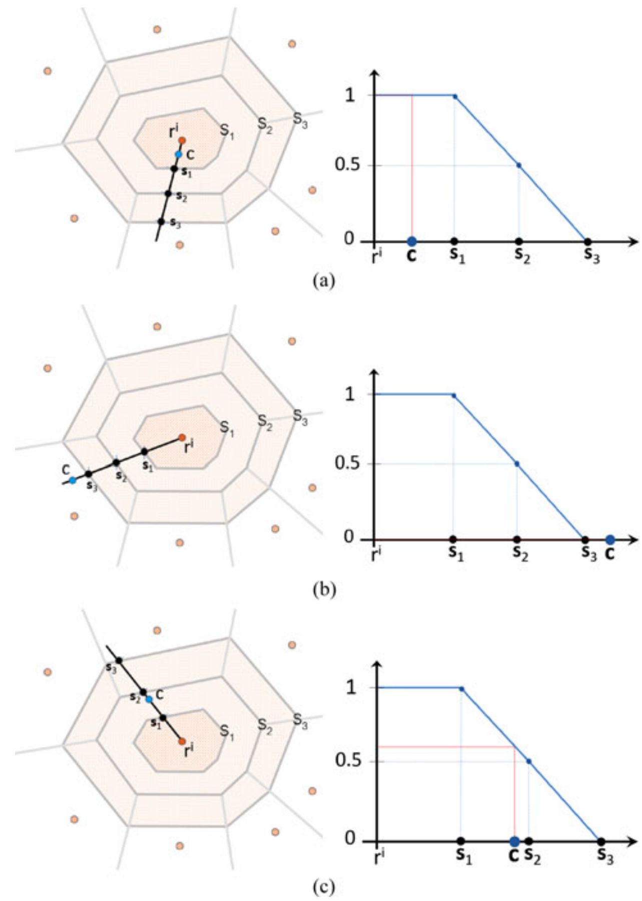


Fig. 3. Bidimensional example of the calculus of membership degrees of a vector $\mathbf{c}$ to a fuzzy model $\widetilde{C}_i$ whose representative is $\mathbf{r}^i$. (a) Example where (6) is applied. (b) Example where (7) is applied. (c) Example where (8) is applied.

4) Illustrating Visually Our Approach: In order to illustrate visually some of the ideas of our approach to learn fuzzy colors, we will use two different examples based on a positive $(\mathbf{r})$ and some negative prototypes $(\mathbf{nr_1}, \ldots, \mathbf{nr_{12}})$, which are represented graphically in Fig. 4. For a better understanding of our proposal, we have developed an example based on an abstract conceptual space in 2-D (prototypes are randomly selected) without any particular interpretation. The other example is based on a conceptual space in 3-D comprised of the RGB with the Euclidean distance (prototypes correspond to colors). In particular, Fig. 4(a) shows the result of obtaining the 0.5-cut for a collection of prototypes in 2-D and 3-D, respectively. The (crisp) Voronoi cells V^i corresponding to the positive prototype $\mathbf{r}^i$ are highlighted in both cases. Fig. 4(b) and (c) shows the core (1-cut) and support of the examples mentioned before in 2-D and 3-D, respectively. The core and the support are obtained by performing a scaling of the Voronoi cell centered on $\mathbf{r}^i$ according to the methodology detailed in Section IV-B2b using $\lambda = 0.5$ and $\lambda' = 1.5$, respectively. Finally, Fig. 4(d) illustrates the core, 0.5-cut, and support for the positive prototype $\mathbf{r}^i$ in 2-D and 3-D, respectively. Membership degrees are obtained by interpolation as we detailed in Section IV-B3.

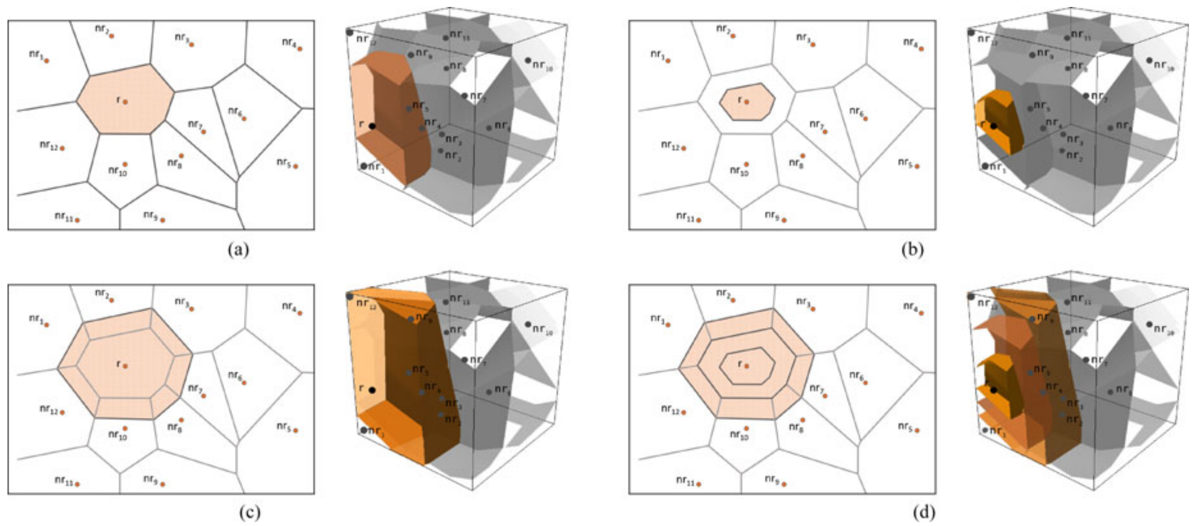


Fig. 4. Illustrative example of our approach in 2-D and 3-D from a positive ($\mathbf{r}$) and some negative prototypes ($\mathbf{nr_1}, \ldots, \mathbf{nr_{12}}$). Different color and gray shades have been used for a better visualization of the faces, without any further meaning. (a) Voronoi tessellation for the set of prototypes. The Voronoi cell for the positive prototype, corresponding to the 0.5-cut, is highlighted. (b) Core, (c) support, and (d) core, 0.5-cut, and support.

5) Calculating Fuzzy Color Spaces: Calculating a fuzzy color space $\widetilde{\Gamma} = \{\widetilde{C}_1, \ldots, \widetilde{C}_m\}$ can be done by simply calculating each fuzzy color separately, using the procedure indicated in the previous sections. Usually, all colors in a fuzzy color space will be defined using the same conceptual space (i.e., crisp color space plus metric) and the same strategy for determining the membership functions.

It is possible to obtain different types of fuzzy color spaces $\widetilde{\Gamma}$ depending on the use of positive R_Γ^+ and negative R_Γ^- prototypes and the scaling factors λ and λ'. Thereby, some interesting conditions can be specified for obtaining particular types of fuzzy color spaces. Let $R_\Gamma^+ = \{\mathbf{r}^1, \ldots, \mathbf{r}^m\}$, $R_\Gamma^- = \cup_{i=1}^m R_i^-$ and $1 \leq \lambda + \lambda' \leq 2$.

Proposition 4.1: Let $R_\Gamma^+ \backslash \{\mathbf{r}^i\} \subseteq R_i^-\ \forall 1 \leq i \leq m$. Then, $\widetilde{\Gamma}$ is a disjoint space.

Proof: Immediate, since $1 \leq \lambda + \lambda' \leq 2$ implies that the intersection of the core of a fuzzy color and the cores of the fuzzy colors corresponding to the negative prototypes is empty. ■

Proposition 4.1 allows us to obtain fuzzy color spaces that represent exclusive color concepts (which does not preclude that they may have fuzzy boundaries with other colors).

Proposition 4.2: Let $R_i^- \subseteq R_\Gamma^+ \backslash \{\mathbf{r}^i\}\ \forall 1 \leq i \leq m$. Then, $\widetilde{\Gamma}$ is a covering space.

Proof: Immediate, since all negative prototypes are also positive prototypes; therefore, there will be no subset of the space that has membership zero to all fuzzy colors. ■

Proposition 4.2 allows us to obtain fuzzy color spaces that ensure that all crisp colors pertain to at least one color category with degree greater than 0.

Proposition 4.3: Let $R_i^- = R_\Gamma^+ \backslash \{\mathbf{r}^i\}\ \forall 1 \leq i \leq m$. Then, $\widetilde{\Gamma}$ is a partition space.

Proof: Immediate from Propositions 4.1 and 4.2. ■

Proposition 4.3 allows us to obtain a fuzzy color space by specifying the set of positive prototypes only, provided that the set of color categories can be represented in a partition space. This is the case considered, for instance, in the definition of basic color terms, as we shall see in the next section.

V. Experimental Results

As we have mentioned, our methodology to the automatic design of customized fuzzy color spaces is based on a collection of crisp colors (each one fully representative of a certain color term). The following examples about the definition of fuzzy color spaces are based on crisp colors and color names provided by the well-known ISCC-NBS system [51] and a collection of crisp colors provided by users. On the basis of these examples, we will analyze the behavior of partition (see Section V-A), noncovering (see Section V-B), and nondisjoint (see Section V-C) fuzzy color spaces. Furthermore, user evaluations in the task of color descriptions and a comparative with other color modeling approaches will also be analyzed (see Section V-D).

In all illustrative examples, the conceptual space RGB plus the Euclidean distance is used. For building fuzzy color spaces, we will define the membership functions of the fuzzy colors on the basis of three volumes $\mathcal{V} = \{V_1, V_2, V_3\}$, with $\Omega = \{1, 0.5, 0\}$.

Additionally, we have developed the website http://www.jfcssoftware.com including all examples and experimental results, together with the software implementing our models.

A. Partition Fuzzy Color Space Example

For defining a fuzzy color space, the set $R_\Gamma^+ = \{\mathbf{r}^1, \ldots, \mathbf{r}^m\}$ of positive prototypes (representatives of the color terms we want to model) and the set $R_\Gamma^- = \bigcup_{i=1}^m R_i^-$ of negative prototypes are needed. In partition fuzzy color spaces, for each color term C_i whose representative is $\mathbf{r}^i$, the set R_i^- can be defined as $R_i^- = R_\Gamma^+ \backslash \{\mathbf{r}^i\}\ \forall 1 \leq i \leq m$; therefore, only the set of positive prototypes is required. In this example, we illustrate our proposal by means of the construction of several partition fuzzy color spaces using the crisp colors provided by the ISCC-NBS system (see Section V-A1) and a collection of colors provided by users (see Section V-A2).

1) Fuzzy Color Spaces Based on the ISCC-NBS: The ISCC-NBS system is based on the pioneering work of Berlin and Kay [34] about color naming and has been tested with humans on a task of color description. In addition, the ISCC-NBS provides three color sets (pairs of linguistic term and crisp color) with different levels of color description, which are collected in the Universal Language of Color [52].

1) *Basic set*: Thirteen color names corresponding to ten basic color terms (pink, red, orange, yellow, brown, olive, green, yellow–green, blue, and purple) and three achromatic ones (white, gray, and black).
2) *Extended set*: Thirty-one color names corresponding to those of the basic set and some combinations of them formed by adding the − ish suffix (Brownish Orange, Purplish Blue, among others).
3) *Complete set*: Two hundred and sixty-seven color names obtained from the extended set by adding five tone modifiers for lightness (very light, light, medium, dark, and very dark) and four adjectives for saturation (grayish, moderate, strong, and vivid). Also, three additional terms substituting certain lightness–saturation combination (pale for light grayish, brilliant for light strong, and deep for dark strong).

Considering these sets as positive prototypes we have defined three partition fuzzy color spaces, named $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$, $\widetilde{\Gamma}_{\mathrm{ISCC-extended}}$, and $\widetilde{\Gamma}_{\mathrm{ISCC-complete}}$. Note that the colors provided by the ISCC-NBS have an exclusive nature (i.e., if a crisp color belongs to one color term, usually, it does not belong to another color term); therefore, the most suitable typology to model the nature of these colors is a partition (disjoint and covering) fuzzy color space.

In order to illustrate and simplify the visualization of the construction process, only the fuzzy color space $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$, which is composed of 13 colors, is shown (the construction process of the rest of spaces is analogous).

Fig. 5 shows the centroids provided by the ISCC-NBS as crisp colors and the volumes associated to the fuzzy colors yellow, blue, green, and gray from $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$ (for the sake of clarity, in the paper, not all the fuzzy colors in $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$ have been shown, but they can be visualized at http://www.jfcssoftware.com thanks to the color naming functionality and the demonstration videos). Fig. 5(b) shows the surfaces of the volumes V_2^i, i.e., the Voronoi cell boundaries, associated with these fuzzy colors. From V_2^i, the core boundaries, i.e., the surfaces of the volume V_1^i, are obtained by means of an uniform scaling with scale factor $\lambda = 0.5$ [see Fig. 5(c)]. In the same way, the support boundaries, i.e., the surfaces of the volume V_3^i, are obtained by means of an uniform scaling with scale factor $\lambda' = 1.5$. In addition, for the fuzzy color yellow, a view of a "truncated" version of the surfaces is shown in Fig. 5(d), where the faces in the sides of the RGB cube have been eliminated to highlight how the surfaces are scaled versions of the Voronoi cell boundary.

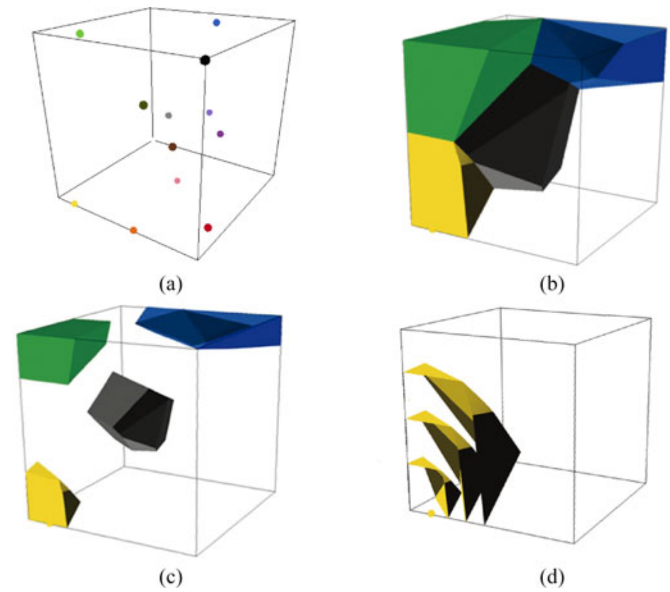
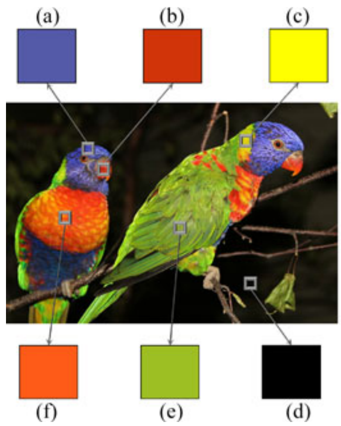


Fig. 5. Centroids provided by the ISCC-NBS and volumes associated to the fuzzy colors Yellow, Blue, Green, and Gray from the $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$. (a) Color centroids in RGB. (b) Volumes of the Voronoi cells: V_2^i. (c) Volumes of the cores: V_1^i. (d) Superimposed views of a "truncated" version of the volumes V_1^i, V_2^i, V_3^i for the fuzzy color Yellow.

Fig. 6 shows the degrees of correspondence of the color of the regions (A)–(F) with fuzzy colors in the three ISCC-NBS-based fuzzy color spaces. Depending on the space, descriptions in terms of linguistic labels with different level of detail are obtained. For example, in $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$, the color (A) corresponds to the fuzzy colors Purple, Blue, and Gray, and the color (B) corresponds to the fuzzy colors Red and Orange, in both cases to some degree. However, descriptions obtained for (A) and (B) by $\widetilde{\Gamma}_{\mathrm{ISCC-extended}}$ and $\widetilde{\Gamma}_{\mathrm{ISCC-complete}}$, which are composed of more colors and are more specific spaces, correspond to only one fuzzy color (Purplish Blue or Reddish Orange, respectively). On the other hand, colors (C)–(F) are recognized as pertaining to the core of fuzzy colors using $\widetilde{\Gamma}_{\mathrm{ISCC-basic}}$, and hence, they are described by a single linguistic label; however, these descriptions are refined by the much more specific spaces $\widetilde{\Gamma}_{\mathrm{ISCC-complete}}$ and $\widetilde{\Gamma}_{\mathrm{ISCC-extended}}$ in most of the cases, in the sense that they correspond to several fuzzy colors to some degree. In our opinion, in these cases, partition spaces provide linguistic descriptions in accordance with human descriptions.

2) Partition Fuzzy Color Space Based on Colors Provided by Users: In order to illustrate our methodology for building customized fuzzy color spaces for a specific context, we shall use a collection of crisp colors provided by users. For obtaining the collection, we have developed a user experiment to collect crisp colors associated to names of fruits. Specifically, users can select colors they consider to be representative of a color term associated with a fruit, and they assign it manually a customized color name from the image collection. The image collection contains examples of fruits, under various conditions of lighting, saturation, etc. The image collection and all users data can be downloaded at http://www.jfcssoftware.com.



Region	RGB Value	$\widetilde{\Gamma}_{ISCC-basic}$ (13 colors)	$\widetilde{\Gamma}_{ISCC-extended}$ (31 colors)	$\widetilde{\Gamma}_{ISCC-complete}$ (267 colors)
A	[84, 90, 167]	0.55 / Purple	1.0 / Purplish Blue	1.0 / Strong Purplish Blue
		0.44 / Blue		
		0.12 / Gray		
B	[209, 52, 9]	0.61 / Red	1.0 / Reddish Orange	1.0 / Vivid Reddish Orange
		0.39 / Orange		
C	[254, 250, 2]	1.0 / Yellow	1.0 / Greenish Yellow	1.0 / Vivid Greenish Yellow
D	[1, 2, 1]	1.0 / Black	1.0 / Black	1.0 / Black
E	[142, 193, 0]	1.0 / Yellow-Green	1.0 / Yellow-Green	1.0 / Strong Yellow-Green
F	[255, 91, 24]	1.0 / Orange	0.67 / Reddish Orange	0.67 / Vivid Reddish Orange
			0.33 / Orange	0.33 / Vivid Orange

Fig. 6. Example showing the behavior of the ISCC-based fuzzy color spaces. Six color samples, corresponding to different color categories, have been selected from the image regions (A)–(D). For the three ISCC-based fuzzy color spaces, the table shows in the last three columns the (nonzero) membership degree of each sample (whose values are shown in the second column) to each fuzzy color. The detail in color description varies depending on the space: the complete one, with more categories, describes the samples with more specific color names, while the basic one use more general terms (implying that some colors, as in (A) and (B), are described as the composition of the primary ones).

Using colors provided by users, we have created a partition fuzzy color space, named $\widetilde{\Gamma}^1_{\text{Fruits}}$ composed by colors associated to the labels banana, blackberry, green apple, lemon, orange, plum, raspberry, red apple and strawberry. We have considered $R^+_\Gamma = \{\mathbf{r}^1, \ldots, \mathbf{r}^9\}$, where each $\mathbf{r}^i$ is an aggregated value from the data provide by the users for each label.

Fig. 7 shows an image composition with several types of fruits, wherein for each fruit, a crisp color has been selected using the color representing the median in a window of 10×10 pixels. Membership degrees of each crisp color to fuzzy colors of $\widetilde{\Gamma}^1_{\text{Fruits}}$ are shown in Fig. 7. Notice that, according to the information provided by users, all crisp colors selected in the image composition are described correctly in terms of fuzzy colors with the label and the appropriate membership degree. Only in the case of color (B), a description with two labels (0.84/lemon and 0.12/banana) is found, which may be not in accordance with human intuition. The problem is the partition typology of the fuzzy color space, which is suitable for color of exclusive nature. On the contrary, the colors lemon and banana are nonexclusive; therefore, other kind of typology is more appropriate. This issue will be taken into account in Section V-C.

In order to illustrate visually the color correspondences between crisp and fuzzy colors, a mapping from the image of Fig. 7 to the fuzzy colors from $\widetilde{\Gamma}^1_{\text{Fruits}}$ is carried out. To compute this mapping, membership degrees of the color of each pixel with the fuzzy colors from $\widetilde{\Gamma}^1_{\text{Fruits}}$ are calculated. Thus, for each fuzzy color $\widetilde{C}_i$, two images are shown: one in gray levels where the membership degrees of each pixel associated with $\widetilde{C}_i$ have been mapped to gray levels between 0 and 255 (white indicates maximum membership and black indicates membership 0 to the fuzzy color), and its color version (calculated by multiplying the representative color $\mathbf{r}^i$ and the corresponding degree of $\widetilde{C}_i$).

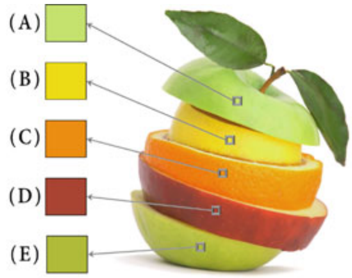
With this mapping operation, the color correspondences between pixels and fuzzy colors are shown visually, and regions corresponding to a color term can be highlighted. In our opinion, this correspondence is reasonably equivalent to what a human would expect. However, in the case of the mapping to red apple, it can be seen that some pixels of the leaves have a membership degree higher than 0 when visually the leaves are not red apple. This is due again to the fact that $\widetilde{\Gamma}^1_{\text{Fruits}}$ is a partition space, so it is a covering space, and it is composed of very few colors; therefore, the fuzzy colors cover all the space, and hence, very different crisp colors have compatibility with the same fuzzy color. This problem will be addressed in Section V-B.

B. Noncovering Fuzzy Color Space Example

A covering fuzzy color space must ensure that every crisp color corresponds at least to one fuzzy color. As a consequence, as we have just seen, in the case of a fuzzy color space consisting of few fuzzy colors, many crisp colors that should not perceptually correspond to any fuzzy color of the space could appear in the support of some fuzzy colors.

For example, Fig. 8 shows an image containing colors that are very different to any of the representatives of the fuzzy color space $\widetilde{\Gamma}^1_{\text{Fruits}}$ defined previously. In this case, correspondences to the color of the regions (A) and (B) with fuzzy colors in $\widetilde{\Gamma}^1_{\text{Fruits}}$ are not consistent with human intuition. Specifically, membership degrees of the region (A) are 0.78/red apple and 0.22/green apple when the region (A) does not perceptually correspond with neither the red nor the green apple. The same applies to region (B), where the membership degree is 1.0/plum. This is because in covering spaces, every crisp color must correspond at least to one color term C_i, and in this case, the set of negative prototypes for each color term C_i is not sufficiently representative of $\neg C_i$.

This last example shows again that the typology of the fuzzy color space should be consistent with the characteristics of the colors we want to model, since otherwise we can get unintuitive results. In this section, a construction of a noncovering fuzzy color space, which will allow us to solve the problem mentioned before, will be illustrated. For this purpose, as in the previous section, we start from a set $R^+_\Gamma = \{\mathbf{r}^1, \ldots, \mathbf{r}^m\}$ of positive prototypes, which are representatives of the color terms we want to model. However, we consider a set of negative prototypes $R^-_i = R^+_\Gamma \setminus \{\mathbf{r}^i\} \bigcup \mathcal{N}_i \quad \forall 1 \leq i \leq m$, where $\mathcal{N}_i$ is a set of additional

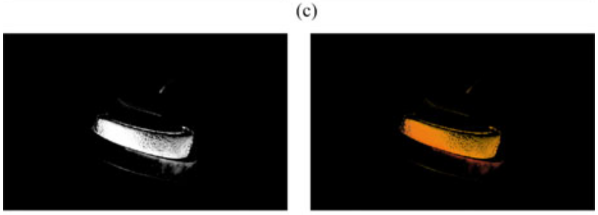


Region	RGB Value	$\widetilde{\Gamma}^1_{Fruits}$ (9 colors)
A	[197, 226, 110]	1.0 / green apple
B	[236, 206, 33]	0.84 / lemon 0.12 / banana
C	[236, 141, 18]	0.91 / orange
D	[169, 68, 50]	1.0 / red apple
E	[178, 189, 60]	1.0 / green apple

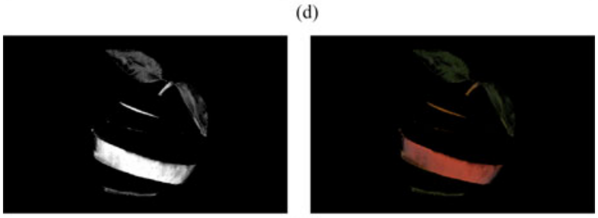
(a)

(b)

(c)



(d)



(e)

Fig. 7. Example of a user-defined fuzzy color space for the fruit color domain. The image shows a heap of six fruit pieces, each with its particular color. (a) For each piece, the table shows a color sample (second column) and its membership degree to the fuzzy colors in $\widetilde{\Gamma}^1_{\texttt{Fruits}}$ (third column). Images (b)–(e) show the mapping of the image to the fuzzy colors green, apple, lemon, orange, and red apple, respectively. In this mapping, the value of each pixel represents its membership degree to the corresponding fuzzy color; a grey level version is shown, where white indicates maximum membership and black indicates membership 0, together with a colored one. (a) Membership degrees of the crisp fruit colors (A)–(E) to the fuzzy colors in $\widetilde{\Gamma}^1_{\texttt{Fruits}}$. (b) Mapping to green apple. (c) Mapping to lemon. (d) Mapping to orange. (e) Mapping to red apple.

Region	RGB Value	$\widetilde{\Gamma}^1_{Fruits}$	$\widetilde{\Gamma}^2_{Fruits}$
A	[156, 92, 54]	0.78 / red apple 0.22 / green apple	*no description*
B	[242, 241, 237]	1.0 / plum	*no description*

Fig. 8. Example comparing covering and noncovering fuzzy color spaces. The image shows two main colors, corresponding to the inside and outside of the coconut, which are not included in the color categories given by the users. Table shows two color samples from the image (second column) and its membership degree to the fuzzy colors in the covering (respectively, noncovering) fuzzy color space $\widetilde{\Gamma}^1_{\texttt{Fruits}}$ (respectively, $\widetilde{\Gamma}^2_{\texttt{Fruits}}$). In the covering case, a forced output is obtained, which is not consistent with human intuition; in the noncovering case, none of the sample colors are compatible with any fuzzy color.

negative prototypes for the color term C_i, which are sufficiently representative of $\neg C_i$.

The set of additional negative prototypes $\mathcal{N}_i$ could be obtained requesting additional information to users. However, since our goal is to obtain adequate models from the minimum information given by users, in this example, we propose an automatic alternative for obtaining this additional set. Specifically, for each color term C_i, an automatic way to define $\mathcal{N}_i$ on the basis of a subsample of the crisp color space is proposed. Given a set $R^+_\Gamma = \{\mathbf{r}^1, \ldots, \mathbf{r}^m\}$ of positive prototypes of the color categories, where $\mathbf{r}^i \in R^+_\Gamma$ is the representative of the color term C_i, we consider a set $\mathcal{M}^k = \{\mathbf{p}_1, \ldots, \mathbf{p}_k\}$ consisting of k points regularly distributed throughout the crisp space (for instance, defining points so that the distance between two points in $\mathcal{M}^k$ is the same in every dimension of RGB). Then, the set of negative prototypes associated with C_i will be $R^-_i = R^+_\Gamma \setminus \{\mathbf{r}^i\} \bigcup \mathcal{N}_i$, where $\mathcal{N}_i$ is calculated from $\mathcal{M}^k$ as follows:

$$\mathcal{N}_i = \{\mathbf{p}_1, \ldots, \mathbf{p}_z\} = \{\mathbf{p}_i \in \mathcal{M}^k, d(\mathbf{p}_i, \mathbf{r}) > d_k \forall \mathbf{r} \in R^+_\Gamma\}$$

where $d(\mathbf{p}_i, \mathbf{r})$ is the distance between $\mathbf{p}_i$ and $\mathbf{r}$, and d_k is the minimum distance between two points in $\mathcal{M}^k$.

According to these considerations, $\widetilde{\Gamma}^2_{\texttt{Fruits}}$ (a noncovering fuzzy color space) is defined. Specifically, we have used the same positive prototypes of the previous $\widetilde{\Gamma}^1_{\texttt{Fruits}}$ fuzzy color space, and for each color term C_i, we have calculated an additional set of negative prototypes $\mathcal{N}_i$, on the basis of 125 colors that are equidistant in every dimension of RGB between them and with the cube boundaries ($\mathcal{M}^{125}$), and following the above equation. Now, with the noncovering fuzzy color space $\widetilde{\Gamma}^2_{\texttt{Fruits}}$, correspondences of the crisp colors (A) and (B) of the Fig. 8 with the fuzzy colors are not conflicting with human intuition. In this case, "no description" is obtained since colors (A) and (B) are not compatible with any fuzzy color in the space, which is consistent with human intuition.

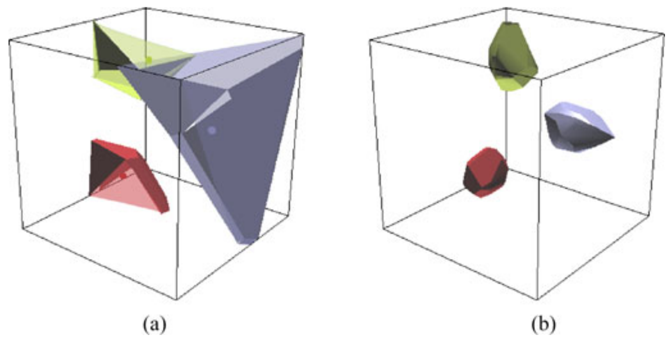


Fig. 9. Core surfaces associated to fuzzy colors red apple, green apple, and plum in the $\widetilde{\Gamma}^1_{\text{Fruits}}$ and the $\widetilde{\Gamma}^2_{\text{Fruits}}$ fuzzy color spaces. (a) $\widetilde{\Gamma}^1_{\text{Fruits}}$ (covering). (b) $\widetilde{\Gamma}^2_{\text{Fruits}}$ (noncovering).

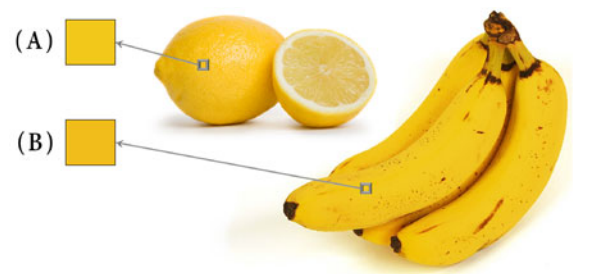
For a better understanding of this problem, Fig. 9 shows an example of the core of the fuzzy colors red apple, green apple, and plum in the $\widetilde{\Gamma}^1_{\text{Fruits}}$ (covering) and in the $\widetilde{\Gamma}^2_{\text{Fruits}}$ (noncovering) fuzzy color spaces. As can be observed, the cores of the fuzzy colors in the $\widetilde{\Gamma}^1_{\text{Fruits}}$ space are formed by many more crisp colors than in the $\widetilde{\Gamma}^2_{\text{Fruits}}$ (volumes are bigger, as in the case of the plum color); therefore, some crisp colors are very distant of the representative of the fuzzy color, so they are perceptually very different. However, because they are situated in the crisp space inside the core, they are compatible with fuzzy colors providing not adequate perceptually results in terms of color descriptions. This problem does not occur in the noncovering space $\widetilde{\Gamma}^2_{\text{Fruits}}$ (volumes of the cores are smaller).

C. Nondisjoint Fuzzy Color Space Example

A disjoint fuzzy color space can represent and model color terms with exclusive nature, not allowing a same crisp color to belong to the core of two or more fuzzy colors (the cores of fuzzy colors have empty intersection). However, in many applications, there are color terms of a nonexclusive nature. For example, following the last examples about colors associated to fruits, a crisp color can be assigned to both banana and lemon with membership degree 1.

This situation is illustrated in Fig. 10, where we have selected two color region whose crisp colors are situated between the two cores of the banana and lemon fuzzy colors in $\widetilde{\Gamma}^1_{\text{Fruits}}$. In the description of the color regions (A) and (B) by means of the $\widetilde{\Gamma}^1_{\text{Fruits}}$ (disjoint), membership degrees of the regions to both fuzzy colors banana and lemon are 0.55 and 0.45, respectively (the membership degree is divided between the two colors). However, a user might say that the membership degree of the color (A) and (B) should be 1 to both the banana and the lemon fuzzy colors because he/she considers that the color terms "lemon" and "banana" are nonexclusive. This example demonstrates again that the typology of the fuzzy color space should be consistent with the nature of the colors we want to model.

In this section, we solve this problem by means of the construction of a nondisjoint fuzzy color space. For this, as in the previous section, we start from a set $R^+_\Gamma = \{\mathbf{r}^1, \ldots, \mathbf{r}^m\}$ of positive prototypes, which are representatives of the color terms we



Region	RGB Value	$\widetilde{\Gamma}^1_{Fruits}$	$\widetilde{\Gamma}^3_{Fruits}$
A	[242, 200, 27]	0.55 / lemon	1.0 / lemon
		0.45 / banana	0.90 / banana
B	[241, 189, 26]	0.51 / banana	0.91 / lemon
		0.49 / lemon	0.78 / banana

Fig. 10. Example comparing disjoint and nondisjoint fuzzy color spaces. The image shows two main colors, corresponding to the lemon and the banana (both included in the color categories given by the users). The table shows two color samples from each fruit (second column) and its membership degree to the fuzzy colors in the disjoint (respectively, nondisjoint) fuzzy color space $\widetilde{\Gamma}^1_{\text{Fruits}}$ (respectively, $\widetilde{\Gamma}^3_{\text{Fruits}}$). In the disjoint case, the membership degrees are "scattered" across both fuzzy colors; in the nondisjoint case, membership degrees are 1 or near to 1, which is more consistent with human intuition.

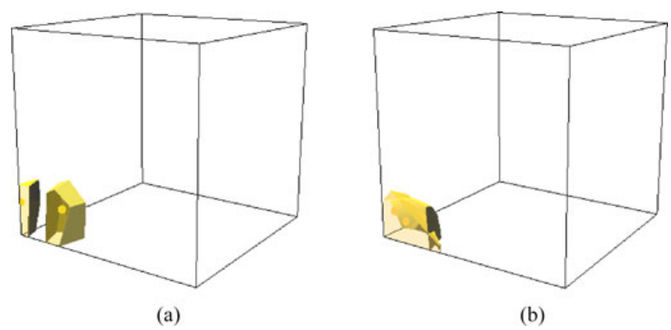


Fig. 11. Core surfaces associated with the fuzzy colors banana and lemon in $\widetilde{\Gamma}^1_{\text{Fruits}}$ and in $\widetilde{\Gamma}^3_{\text{Fruits}}$. (a) $\widetilde{\Gamma}^1_{\text{Fruits}}$ (disjoint). (b) $\widetilde{\Gamma}^3_{\text{Fruits}}$ (nondisjoint).

want to model, and a set $R^-_\Gamma = \bigcup_{i=1}^m R^-_i$ of negative prototypes, such as for each color term C_i a set $R^-_i = \{\mathbf{nr}^1, \ldots, \mathbf{nr}^k\}$ corresponding to colors in $\neg C_i$.

In particular, we have defined $\widetilde{\Gamma}^3_{\text{Fruits}}$ (a nondisjoint fuzzy color space) considering the same set of positive prototypes used in the previous examples. However, in this case, for each color term, we have used a set of negative prototypes provided by a user. Additionally, the set of negative prototypes is also extended according to the approach mentioned in the previous section by means of a subsample of the crisp space; therefore, $\widetilde{\Gamma}^3_{\text{Fruits}}$ is a nondisjoint and noncovering fuzzy color space. Hence, membership degrees of the regions (A) and (B) in Fig. 10 to both fuzzy colors banana and lemon from $\widetilde{\Gamma}^3_{\text{Fruits}}$ are 1 or near to 1, just as a user who considers a nonexclusive nature of the colors would suggest.

In order to illustrate visually the difference between disjoint and nondisjoint spaces, Fig. 11 shows an example of the color terms banana and lemon modeled by a disjoint and a nondisjoint space. In particular, the cores of the fuzzy colors equivalent to banana and lemon in the fuzzy color spaces $\widetilde{\Gamma}^1_{\text{Fruits}}$ (disjoint) and $\widetilde{\Gamma}^3_{\text{Fruits}}$ (nondisjoint) are shown, respectively. As can be observed, the cores of the fuzzy colors do not overlap in the disjoint case, while in the nondisjoint case, the cores are overlapped. Note that the fuzzy colors are different in both spaces.

D. Validation With Users and Comparison With Other Models

In this section, the goodness of our fuzzy color spaces will be analyzed on the basis of psychophysical experiments with subjective assessments. Many psychophysical experiments related to color modeling, based on users assigning color names to color stimuli, can be found in the literature, as we have seen. However, most of them do not consider the fuzzy nature of the problem, only providing pairs of data (crisp color and color name) without associated degree; therefore, they are not suitable for our purpose. Hence, we have developed an experiment in which the aim is to analyze to which extent the learned fuzzy color space match the semantics of the color terms as understood by the user. We study this matching that defines the goodness of a fuzzy color space by studying the difference between the compatibility crisp color, fuzzy color assessed by a user, and the membership degree of the crisp color to the fuzzy color as provided by the fuzzy color space.

For this purpose, we have developed several experiments with 30 users (15 men and 15 women), and we have set up a context where most of the users are handy (fruit colors). The setting for our different experiments is the following.

1) We asked each user to define a collection of color terms for the fruit context, and to provide prototypes for each of them by choosing representative pixels in fruit images from a provided database. Users have provided around ten color terms, as well as one positive and several negative prototypes for each color term. All users have made the experiment using a color calibrated monitor under the same illumination conditions. The provided database from fruits image and all data from users can be downloaded at http://www.jfcssoftware.com. With these prototypes, fuzzy color spaces of different typologies are obtained for each user using the methodology provided in this paper, as we shall see later in this section. For a certain user and a particular space typology, let $\widetilde{\Gamma}_u = \{\widetilde{C}_1, \ldots, \widetilde{C}_m\}$ be the fuzzy color space.
2) In order to assess the validity of each $\widetilde{\Gamma}_u$, we ask user u to assess the compatibility between each fuzzy color in $\widetilde{\Gamma}_u$ and several crisp colors randomly selected at different distances from the prototype of the fuzzy color. The set of crisp colors used in the assessment of the fuzzy color space $\widetilde{\Gamma}_u$ by the user u can be represented by an $n \times m$ matrix of crisp colors MP_u, where $\mathbf{c}_{ij}$ is the crisp color j employed in the assessment of the fuzzy color $\widetilde{C}_i$. On average, we have obtained 2000 assessments for each user. It is important to remark that these assessments are not necessary for our learning methodology, but are collected in our experiment just to verify the goodness of the obtained fuzzy color spaces.
3) Contrary to computers, users employ just a few compatibility degrees, usually linguistically expressed. Hence, we have asked users to assess the compatibility between a fuzzy color and a crisp color using a set of five fuzzy assessments FA={"nothing," "few," "medium," "enough," "a lot"}, represented by five triangular membership functions in $[0, 1]$ defining a partition in the sense of Ruspini with cores in 0, 0.25, 0.5, 0.75, and 1, respectively. The $n \times m$ matrix MA_u represents the set of fuzzy assessments given by user u for the fuzzy color space $\widetilde{\Gamma}_u$, where $f_{ij} \in FA$ is the fuzzy assessment of the correspondence between $\mathbf{c}_{ij}$ and $\widetilde{C}_i$ as given by the user u:

$$MP_u = \begin{bmatrix} \mathbf{c}_{11} & \ldots & \mathbf{c}_{m1} \\ \mathbf{c}_{12} & \ldots & \mathbf{c}_{m2} \\ \vdots & \ldots & \vdots \\ \mathbf{c}_{1n} & \ldots & \mathbf{c}_{mn} \end{bmatrix} MA_u = \begin{bmatrix} f_{11} & \ldots & f_{m1} \\ f_{12} & \ldots & f_{m2} \\ \vdots & \ldots & \vdots \\ f_{1n} & \ldots & f_{mn} \end{bmatrix}.$$

4) We define the error between the information provided by a user u and a fuzzy color space $\widetilde{\Gamma}$ on the basis of the fuzzy assessments MA_u as

$$\text{Error}_u^{\widetilde{\Gamma}} = \frac{1}{n \times m} \sum_{i=1}^{m} \sum_{j=1}^{n} \left(1 - f_{ij}\left(\widetilde{C}_i(\mathbf{c}_{ij})\right)\right) \quad (9)$$

where f_{ij} is a fuzzy assessment of MA_u, $\mathbf{c}_{ij}$ is a crisp color of MP_u, $\widetilde{C}_i(\mathbf{c}_{ij})$ is the compatibility degree of $\mathbf{c}_{ij}$ with the fuzzy color $\widetilde{C}_i$ provided by the fuzzy color space $\widetilde{\Gamma}$, and $f_{ij}\left(\widetilde{C}_i(\mathbf{c}_{ij})\right)$ is the matching degree between the membership degree provided by $\widetilde{\Gamma}$ and the fuzzy assessment provided by the user u.
For example, if a user u considers that for a crisp color $\mathbf{c}_{11}$, there is "enough" correspondence with the color term C_i modeled by $\widetilde{C}_1$ (note that in our case, the term "enough" is defined by "around of 0.75"), f_{11} is the triangular function of FA with core in 0.75. However, if the user considers that there is "few" correspondence between $\mathbf{c}_{11}$ and $\widetilde{C}_1$ (the term "few" is defined by "around of 0.25"), the fuzzy assessment f_{11} is the triangular function of FA with core in 0.25. In this way, for all crisp colors of MP_u, the matrix of fuzzy assessments MA_u is obtained. Then, the error between the information provided by a user u and a fuzzy color space $\widetilde{\Gamma}$ on the basis of the fuzzy assessments MA_u is calculated by means of (9).

Using this experimental setting, in the next sections, we show the result of several experiments for assessing the validity of different typologies of fuzzy color spaces obtained with our methodology. We also use the error measure defined above for validating the fuzzy color spaces obtained using other existing techniques in the literature for the fruit context, allowing us to compare the results provided by our techniques with existing approaches.

1) Analyzing Our Fuzzy Color Spaces: In Table I, for each user, the error calculated by (9) is shown. This error is unique for each user and collects how far the information provided by

TABLE I
AVERAGE ERROR AND STANDARD DEVIATION, ON THE BASIS OF EQUATION (9), IN OUR PROPOSALS, NONFUZZY, PARTITION-BASED, AND CLUSTERING-BASED PROPOSALS WITH RESPECT TO THE FUZZY ASSESSMENTS OF 30 USERS

	OUR PROPOSALS			NONFUZZY			PARTITION-BASED				CLUSTERING-BASED	
	DISJOINT		NON	DISJOINT		NON	RGB	RGB	RGB	HSI	FUZZY C-MEANS	
	COVER.	NONCOV.	DISJOINT	COVER.	NONCOV.	DISJOINT	5-5-5	3-3-3	2-2-2	Munsell	$\text{cen}_j = R_\Gamma^+$	$\text{cen}_j = C_m$
User 1	0.1395	0.1378	0.0772	0.2193	0.2085	0.2028	0.4108	0.4527	0.5016	0.4405	0.1407	0.119
User 2	0.2413	0.2069	0.1174	0.3315	0.3024	0.2961	0.4086	0.447	0.5269	0.4462	0.2528	0.2508
User 3	0.2172	0.2058	0.0923	0.322	0.2941	0.2767	0.3838	0.4198	0.5009	0.4226	0.2565	0.2753
User 4	0.1534	0.1511	0.0652	0.2161	0.2063	0.1843	0.4084	0.405	0.52	0.4097	0.1512	0.1546
User 5	0.222	0.2129	0.1104	0.3242	0.2981	0.2894	0.4129	0.397	0.4718	0.4292	0.2315	0.264
User 6	0.2238	0.2181	0.1098	0.3314	0.2987	0.2999	0.4294	0.4292	0.513	0.4594	0.2391	0.269
User 7	0.2236	0.2164	0.0836	0.3291	0.3007	0.2706	0.3943	0.4296	0.4778	0.4079	0.257	0.2615
User 8	0.2118	0.2079	0.128	0.309	0.2742	0.2642	0.3832	0.3948	0.4757	0.4239	0.2362	0.2708
User 9	0.2349	0.2281	0.0952	0.3223	0.3054	0.2833	0.41	0.4551	0.5267	0.4235	0.2465	0.2799
User 10	0.2516	0.2363	0.1329	0.3373	0.2725	0.2493	0.313	0.4235	0.536	0.3167	0.2852	0.2876
User 11	0.2253	0.2021	0.1078	0.3234	0.3004	0.2939	0.4284	0.4105	0.487	0.4318	0.2483	0.2751
User 12	0.2283	0.2096	0.1195	0.3244	0.2987	0.2924	0.3996	0.439	0.4827	0.4137	0.2485	0.246
User 13	0.2203	0.2199	0.106	0.3176	0.2962	0.2939	0.3784	0.4226	0.5073	0.4419	0.2181	0.2528
User 14	0.2265	0.2077	0.1221	0.324	0.2949	0.2888	0.3825	0.4261	0.4888	0.4139	0.2357	0.2597
User 15	0.2136	0.2082	0.0992	0.3278	0.2993	0.2951	0.3864	0.4012	0.5073	0.4271	0.234	0.2559
User 16	0.2211	0.2099	0.1126	0.3211	0.2916	0.2887	0.3738	0.4028	0.4831	0.43	0.2221	0.2772
User 17	0.2262	0.2095	0.0964	0.3205	0.2887	0.2725	0.3769	0.4112	0.4835	0.4154	0.2474	0.2654
User 18	0.1987	0.1914	0.102	0.3208	0.3024	0.2972	0.3973	0.4165	0.4978	0.4438	0.2339	0.2689
User 19	0.2339	0.2316	0.0768	0.326	0.3046	0.2778	0.3976	0.4179	0.5035	0.4135	0.2341	0.2418
User 20	0.2328	0.2152	0.116	0.324	0.295	0.2887	0.4084	0.4271	0.4766	0.4444	0.2455	0.2589
User 21	0.2574	0.2494	0.1472	0.3383	0.2862	0.2836	0.3723	0.4511	0.5418	0.3923	0.268	0.3001
User 22	0.2286	0.2187	0.1147	0.3302	0.3016	0.2933	0.4142	0.43	0.4899	0.4492	0.2547	0.2655
User 23	0.2327	0.2272	0.1085	0.3209	0.2927	0.2869	0.3822	0.4224	0.4885	0.4027	0.2242	0.2269
User 24	0.2287	0.215	0.1027	0.3205	0.2858	0.2834	0.402	0.425	0.4795	0.4118	0.2381	0.2552
User 25	0.2198	0.2146	0.108	0.3248	0.2993	0.2964	0.4082	0.4123	0.4801	0.4352	0.2156	0.2337
User 26	0.2318	0.2211	0.0966	0.329	0.3034	0.3006	0.4228	0.4151	0.4867	0.4406	0.2086	0.2373
User 27	0.2183	0.2013	0.1032	0.3312	0.306	0.3006	0.4057	0.4125	0.4953	0.4347	0.2259	0.2409
User 28	0.232	0.2088	0.1117	0.3249	0.2946	0.2928	0.394	0.4302	0.4959	0.4451	0.2422	0.106
User 29	0.233	0.2223	0.0955	0.3292	0.3018	0.297	0.3821	0.4132	0.5227	0.4291	0.2215	0.238
User 30	0.2245	0.2142	0.1286	0.3278	0.2945	0.2899	0.3955	0.4121	0.5055	0.424	0.2263	0.2561
Average	0.2218	0.2106	0.1062	0.3183	0.2899	0.281	0.3954	0.4218	0.4985	0.424	0.2330	0.2465
St. Dev.	0.0234	0.0213	0.0174	0.0279	0.0238	0.0265	0.0222	0.0158	0.0190	0.0255	0.0287	0.0443

the user and by the fuzzy color space is in the task of assigning color stimulus (crisp colors) to color names.

In our comparison experiments, we wanted to emphasize the goodness of our approach in a domain of color terms that suggests a typology of nondisjoint and noncovering space, which is not usually considered by other techniques. Results over domains compatible with partition spaces are similar to those obtained with our approach, showing that we can deal with spaces of any of these kinds.

In our experiments for determining the goodness in relation to user's perception, and due to the nature of the selected domain (fruits colors), the partition spaces (disjoint and covering) of the kind provided by most techniques in the literature have yielded worst results. As expected, results are better in the case of disjoint and noncovering spaces, while the spaces that yield the best results according to users are the nondisjoint and noncovering spaces. In particular, the average error obtained for nondisjoint and noncovering fuzzy color spaces is 0.1062 against 0.2106 for disjoint and noncovering and, 0.2218 for partition fuzzy color spaces. Again, this shows that there are different kinds of spaces according to the specific application and gives value to our technique as it is able to adapt to the characteristics of the context and users.

2) Comparison With Nonfuzzy Models: In this section, we have compared the results of our fuzzy models with models obtained through the paradigm of conceptual spaces without fuzzification, i.e., we have used as color model each color cell obtained from the Voronoi tessellation. In the case of the partition spaces, a quantization is obtained. In the case of nondisjoint or noncovering models, we obtain a set of cells that may either overlap or not cover the whole space. Our goal has been to take as nonfuzzy examples the more similar models to our fuzzy models so that the comparison was as fair as possible.

Table I shows, besides the error obtained in our proposed fuzzy color spaces, the error obtained by nonfuzzy approaches. It can be observed that our models also give good results with respect to nonfuzzy models, as expected since colors are better modeled by fuzzy sets.

3) Comparison With Models Based on a Fuzzy Partitioning: These models are based on a partition of the space by means of trapezoidal functions along the color components. They do not require experimentation to learn the parameters of the functions and obviously, only partition spaces are allowed. In this type of models, membership degrees of crisp colors to fuzzy colors are usually obtained by means of an aggregation of the membership degree of each color component by means of a t-norm.

In order to reproduce these models (based on a fuzzy partitioning), for each user, we have defined different fuzzy color spaces using equidistributed trapezoidal functions. On the one hand, we have created three fuzzy color spaces based on equidistributed trapezoidal functions defined over RGB color components. Particularly, the $\widetilde{\mathrm{RGB}}_{2,2,2}$ space with eight colors defined by two trapezoidal functions over each RGB color component, the $\widetilde{\mathrm{RGB}}_{3,3,3}$ space with 27 colors defined by three trapezoidal functions and the $\widetilde{\mathrm{RGB}}_{5,5,5}$ with 125 colors defined by five trapezoidal functions. And on the other hand, we have also used a fuzzy color space $\widetilde{\mathrm{HSI}}_{\mathrm{Munsell}}$ based on the Munsell space, defined by trapezoidal functions over HSI color components, we have proposed in a previous work [13]. We have calculated the membership degree of a crisp color $\mathbf{c}$ to a fuzzy color $\widetilde{C}_i$, whose representative is $\mathbf{r}^i$ by means of an aggregation operation using the minimum as t-norm and the trapezoidal functions to $\mathbf{r}^i$ in each color component. For the lack of space, trapezoidal functions for each fuzzy color spaces can be viewed at http://www.jfcssoftware.com.

In Table I, besides the error obtained with our proposals, for each user, the error using models based on a partitioning is also shown. In this case, the results are worse, in all cases, than those obtained with our methodology, since with these models, the irregular nature of customized colors defined by users is not contemplated. In addition, the results are even worse than those obtained by nonfuzzy approaches, since with these models, the number of colors to consider is not the same as those defined by a user because of the regular distribution of the membership functions. Specifically, we have obtained an average error of 0.3954 in the best of cases; therefore, we remark again that the typology of fuzzy color space should be consistent with the nature of the colors a user want to model.

4) Comparison With Models Based on Clustering Techniques: In this section, we have considered the clustering algorithm fuzzy c-means used by several authors as clustering-based color model [31], [43], [53]. The membership function is defined as

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{C}\left(\frac{d(x_i,c_j)}{d(x_i,c_k)}\right)^{\frac{2}{m-1}}}$$

where $d(x_i, c_j)$ is the distance of the color x_i to the cluster whose centroid is c_j, and $d(x_i, c_k)$ is the distance of the color x_i to other of the clusters whose centroid is c_k. The parameter m provides a degree of "fuzzification."

In this case, for each user, two kinds of spaces based on the fuzzy c-means are defined: one of them considering as centroids the set R^+_Γ provided by the users, and the other considering as centroids those obtained from the clustering process calculated with all positive prototypes provided by all users for all color terms. Furthermore, the experiment has been performed for all integer values of the parameter m between 1 and 30 (which are common values used in the literature). However, Table I only shows results with $m = 2$ because the minimum error is obtained with them.

As we can see in Table I, results considering the set R^+_Γ as centroids are slightly better than considering the centroids from the clustering process. This is because in the first case, for each color, the centroid is the representative $\mathbf{r}^i$, which is calculated independently with the positive prototypes that users provide, while in the second case, the centroids are calculated considering all data provided by the users; therefore, higher dispersion and less representativeness is contemplated. In addition, the error obtained by our proposals is lower than that obtained by clustering, since the evolution of the membership degrees in models based on clustering has a spherical behavior, while, when a user defines customized colors, the color model usually has an irregular form. In particular, we have obtained an average error of 0.2330 in the best of the clustering-based models versus an error of 0.2218 in the worst case of our proposal.

VI. Conclusion

In this paper, we have introduced formal definitions of the concepts of "fuzzy color" and "fuzzy color space." We have also proposed different types of fuzzy color spaces (partition, disjoint, and covering), which allow us to address the subjectivity, context dependence, and imprecision in color modeling.

In addition, we have established a methodology for modeling the semantics of color terms in different contexts, by using a suitable collection of prototypes. With our methodology, not only the 11 basic colors of Berlin and Kay [34], but any color term for a particular application, context, and/or individual user can be modeled. Tedious experiments and/or a large number of observers are not necessary, since the only information required is a representative crisp color for each linguistic color term to be modeled. Moreover, no regular forms are imposed in our color models, contrary to what happens with some of the most used approaches in the literature.

Our models are based on the paradigm of conceptual spaces [23], [24]. However, we have extended this paradigm (that only permits to obtain crisp quantizations) in order to obtain fuzzy colors and different typologies of fuzzy color spaces. In particular, we have proposed a new methodology to "fuzzify" the Voronoi tessellations in terms of the metric defined in the conceptual space (particularly the Euclidean distance), such that the membership degree decreases in a nonuniform fashion as the distance from the prototype increases. Furthermore, the use of positive and negative prototypes in conceptual spaces is another original contribution, which has removed some limitations of conceptual spaces. Particularly, the use of positive and negative prototypes allows us to model colors independently of each other (in conceptual spaces, in order to learn a color term, other color terms are needed). This characteristics allows us additionally to obtain the above-mentioned different typologies of fuzzy color spaces (not restricting to fuzzy partition spaces, as it is the case of existing models in the literature).

The proposal has been illustrated by building fuzzy color spaces in RGB on the basis of the well-known ISCC-NBS color naming system, as well as fuzzy color spaces based on collections of colors provided by users. Some illustrative examples have also been elaborated in order to show the suitability of our fuzzy color spaces for modeling colors with different semantic interpretations. These examples show that the typology of the fuzzy color space should be consistent with the characteristics of the colors we want to model, since otherwise we can get nonintuitive results. Furthermore, evaluations of the models by comparing them with subjective assessments of users and a comparison with other approaches for color modeling have also been carried out, obtaining satisfactory results.

Finally, the proposed models have been implemented, and the software has been made available to the scientific community at http://www.jfcssoftware.com. All the experimentation data used in this paper and additional materials in the task of color modeling have also been included at the website.

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