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# TARGET RETURNS WITHIN RISK PROGRAMMING MODELS: A MULTI-OBJECTIVE APPROACH

**J. Berbel***

*An approach to deal with risk in agricultural planning with a safety-first model is presented. The approach, named mean-partial absolute deviation (mean-PAD), is aimed at improving existing safety-first methods by combining an accurate estimation of the probability of failure with multiobjective programming. The logic of the method is explained by a simple example. Although the mean-PAD approach is a multiobjective technique by nature, it can be made operational by using common linear programming codes.*

## Introduction

The best-known risk-return model is the mean-variance (E-V) analysis, first proposed by Markowitz (1952), wherein risk is measured as the variance and return as the mean. Some shortcomings of the E-V analysis are discussed in Holthausen (1981), Fishburn (1977) and Tauer (1983). They argue that variance is an inappropriate measure of risk because decision-makers are interested in 'downside-risk' or in the probability of not achieving a minimum target.

The use of safety-first (SF) models, of the lexicographic type, is a common approach to risk return analysis. A review of alternative formulations of risk constraints in a Linear Programming (LP) model is found in Kennedy and Francisco (1974). The SF approach assumes that the probability of not achieving some critical value of gross margin is a crucial element together with the expected outcome of the decision. Three variants are suggested: (1) minimising the probability of failure subject to a target level of the gross margin; (2) maximising the critical low value of gross margin subject to a specified probability; (3) maximising gross margin subject to some probability of attaining a certain target.

All the SF methods require probability estimates for the lower tail of the relevant distribution. Commonly, this is the probability of returns not achieving a predetermined 'disaster level' which can be estimated by using the Tchebyschev inequality:

* Julio Berbel is a Ph.D. student in the Department of Agricultural Economics, Management and Statistics, University of Cordoba, Apdo 3048, 14080-Cordoba, Spain. Part of the work reported here was done while the author was a postgraduate student at the Department of Agricultural Economics, Manchester University. The help of Professor C. Romero, Dr. N. Russell, Dr. F. Sandiford and an anonymous reviewer is acknowledged. The English editing of Cristine Mendez is also appreciated.

$$\Pr[\,|x - E(x)| > k\,s] \leq 1/k^2 \qquad (1)$$

where E(x) is the expected value of returns, s is the standard deviation, and k is a factor which will determine the probability limit.

The probability bounds obtained by using (1) are highly conservative. When the distribution is known, this probability of failure can be determined more accurately. However, in a farm planning context, it is more realistic to treat the probability distribution as unknown.

Berck and Hihn (1982) introduced a less conservative inequality, based on the semi-standard deviation $s^-$ (square root of the semi-variance):

$$\Pr[\,|x - E(x)| > k\,s^-] \leq 1/k^2 \qquad (2)$$

Atwood (1985) proposes the use of lower-order partial moments to improve the results of the generalised semi-variance inequality (2) and shows that this inequality can be generalised to:

$$\Pr(x<g) = \Pr[x<t-pQ(k,t)] \leq 1/p^k \qquad (3)$$

where g=safety level threshold; t=auxiliary parameter to estimate probability of failure (t>g); p=parameter (positive) and Q(k,t)=k−th root of the moment of order k below t, which is defined as follows:

$$Q(k,t) = \left[ \int_{-\infty}^{t} (t-x)^k f(x)\,dx \right]^{1/k} \qquad (4)$$

**The Mean-PAD Model**

Atwood shows that for three different distribution functions (normal, gamma and non-standardised beta), the probability of failure can be improved greatly by using lower partial moments.

The selection of an appropriate level for t can be difficult in applied research. Atwood proposes a method whereby the least constraining level for t can be selected endogenously by solving a LP model.

The first moment for t, Q(1,t), is easily incorporated into a LP model. The inequality (3) can be particularised for k=1, to estimate the probability of failure. We define g in (3) as follows:

$$g = t - [p * Q(k,t)] \qquad (5)$$

p can be obtained from (5) as:

$$p = (t-g) / Q(k,t) \qquad (6)$$

If we substitute (6) into (3), the result is:

$$\Pr(x<g) \leq [Q(k,t)/(t-g)]^k \qquad (7)$$

and we can work with expression (7), which, for k=1, can be included in a linear programme as:

$$\Pr(x<g) \leqslant Q(1,t)/(t-g) \quad (8)$$

Inequality (8) can be derived using only the definition of partial moment so that Atwood's inequality is not required as shown in the Appendix. We will call Q(1,t) the Partial Absolute Deviation or PAD; this concept is the basis of the proposed model.

## Risk Programming Methods

Barry and Robinson (1975) argue that all the mean-variance approaches implicitly use the Tchebyschev inequality (1) because the decision-maker does not want to minimise variability and the critical parameter is the associated probability of failure. We can extend this argument to all the risk programming approaches.

The main risk-return programming approaches use the mean and a moment associated to risk. This moment Q(k,t) can be classified depending on two parameters: the order k and the level t from which deviations are measured. This point t can be a predetermined value or the mean value. Different risk-return programming approaches treat this as: MOTAD (t=mean, k=1) (Hazell, 1971); target MOTAD (t<mean, k=1) (Tauer, 1983); mean-semivariance (t=mean, k=2) (Markowitz, 1968) and mean-partial squared deviations (t<mean, k=2) (Porter, 1974). The mean variance approach (Markowitz, 1952) is not included because variance is not a moment Q(k,t).

As Fishburn demonstrated, the efficient set generated with Q(k,t) of which target MOTAD is a particular case is a subset of the second stochastic dominance criterion. This characteristic and the fact that target MOTAD can be included in a LP makes the model particularly suitable for agricultural planning.

We may combine this model and the Atwood inequality. This is the basis of the mean-PAD model. Instead of the moment Q(1,t) or PAD, we can work with the probability of failure estimated by (8). If we proceed by this method, t must be fixed and it should be at least as large as g (safety level). When t (the parameter from which deviations are measured) approximates g, the fraction Q/(t−g) tends to infinity so that the probability limit is unbounded.

In our model, the second objective is to minimise the probability of failure, but this criterion is equivalent to the objective of minimising PAD. The probability of failure is computed by (9), which comes from (8) by substituting Q(1,t) for (PAD/m) with m equal to number of years or considered periods:

$$\Pr(x<g) \leqslant PAD/m(t-g) \quad (9)$$

It is important to remember that in Tauer's work t is regarded as the critical value. However, in the SF version, g is crucial and t is only a required parameter.

## Risk-Return in a Multiple Objective Context

Multiple Criteria Decision-Making (MCDM) techniques deal with problems in which more than one criterion is simultaneously considered. Within the MCDM paradigm, one of the most promising approach to deal with risk is

Multiobjective Programming (MOP). These techniques deal with the simultaneous optimisation of several objectives subject to a set of constraints. As the optimum cannot generally be attained simultaneously with different objectives, MOP tries to develop the efficient or nondominated set.

The elements of an efficient set are feasible solutions and must be such that there are no other solutions that can achieve the same or better performance for all the objectives and strictly better for at least one objective (Romero and Rehman, 1984).

MOP has not been used extensively in farm planning. Hitchens *et al.* (1978) analyse the trade-offs between monetary profits and ecological aspects in regional planning in Australia. Vedula and Rogers (1981) applied a similar model in India to the conflict between profits and total irrigated area. Romero *et al.* (1987) use the MOP approach to deal with an agricultural planning problem within an agrarian reform programme in Spain.

The risk-return analysis is based on a two-criteria model, in which the first criterion is the expected value and the second criterion is the probability of failure or the associated PAD. The method tries to develop the efficient set in the two criteria space. The structure of the mean-PAD model is:

$$\begin{aligned} &\underline{\text{Eff}}\,(GM, PAD) \\ &GM = G\,X \\ &PAD = \sum_{i=1}^{m} Pr(i)\,N(i) \end{aligned} \qquad (10)$$

subject to:

$$\begin{aligned} &A\,X \leqslant b \\ &S(i)\,X + N(i) \geqslant t \qquad \text{for } i=1,\ldots,m \end{aligned}$$

where Eff means the efficient set, A=matrix of technical coefficients, GM=gross margin, G=vector of expected gross margins per unit of activity level, X=vector of activity levels, PAD=probability-weighted sum of negative deviation from t for the m years (states of nature), S(i)=vector of gross margin for the m years (states of nature), N(i)=vector of negative deviations, and t is a parameter (scalar).

The efficient set or trade-off curve can be developed by several methods: weighting, constraint, Simplex Multicriteria and Non-Inferior Set Estimation (NISE). A detailed explanation of these methods can be found in Cohon *et al* (1978). The constraint method optimises one objective while the other is treated as a restraint. Parameterising the right-hand sides of the constraint inequalities, the efficient set is approximated. Within a target MOTAD context, both Tauer and Watts *et al* (1984) use this technique. In fact, they maximise returns while parametrically varying the upper limit on deviations.

## A Numerical Illustration

To illustrate some of the concepts discussed above, the mean-PAD model is applied to a small numerical example. Let us assume a 200-hectare farm with four activities and six maximum-type constraints: available area, hours of labour and rotational constraints.

Table 1 presents the tableau of real activities and constraints. The PAD row will give the sum of deviations. In our example, if we assume that the probability of each state of nature is equal to 1/m, we get the first-order moment if we divide PAD by the number of years m (see (9)). The first two rows have a 'Free RHS' which means that they are unconstrained rows (i.e. objectives). The safety level assumed is g=20,000.

**Table 1 Mean-PAD Model for the Example Problem**

| *Row and Unit* | *X1* | *X2* | *X3* | *X4* | *N1* | *N2* | *N3* | *N4* | *N5* | *RHS* |
|---|---|---|---|---|---|---|---|---|---|---|
| Mean GM ($) | 100 | 95 | 140 | 136 | | | | | | FREE |
| PAD ($) | | | | | 1 | 1 | 1 | 1 | 1 | FREE |
| Year 1 ($) | 120 | 100 | 75 | 150 | 1 | | | | | $\geq t$ |
| Year 2 ($) | 110 | 95 | 175 | 100 | | 1 | | | | $\geq t$ |
| Year 3 ($) | 100 | 90 | 200 | 70 | | | 1 | | | $\geq t$ |
| Year 4 ($) | 90 | 100 | 100 | 160 | | | | 1 | | $\geq t$ |
| Year 5 ($) | 80 | 100 | 150 | 200 | | | | | 1 | $\geq t$ |
| Area (ha) | 1 | 1 | 1 | 1 | | | | | | $\leq 200$ |
| Labour (hours) | 1 | 0.5 | 1.6 | 2 | | | | | | $\leq 300$ |
| Rotation (ha) | 1 | | | | | | | | | $\leq 100$ |
| Rotation (ha) | | 1 | | | | | | | | $\leq 100$ |
| Rotation (ha) | | | 1 | | | | | | | $\leq 100$ |
| Rotation (ha) | | | | 1 | | | | | | $\leq 100$ |

Table 2 and the curve in Figure 1 show the trade-offs between risk (measured as the probability of failure) and expected gross margin. The lower value of t which generates non-dominated solutions is t=23,445 as can be found by a parametric analysis of t. We choose a value of t=23,800 as the parameter from which deviations are measured. Table 2 is the efficient set generated by using this value.

**Table 2 Extreme Efficient Points**

| *Point* | *Gross Margin (GM)* | *PAD* | *Pr (GM <20,000)* |
|---|---|---|---|
| A | 24.945 | 738 | 3.88% |
| B | 25.480 | 1.321 | 6.95% |
| C | 25.892 | 3.019 | 15.89% |
| D | 25.960 | 3.500 | 18.42% |

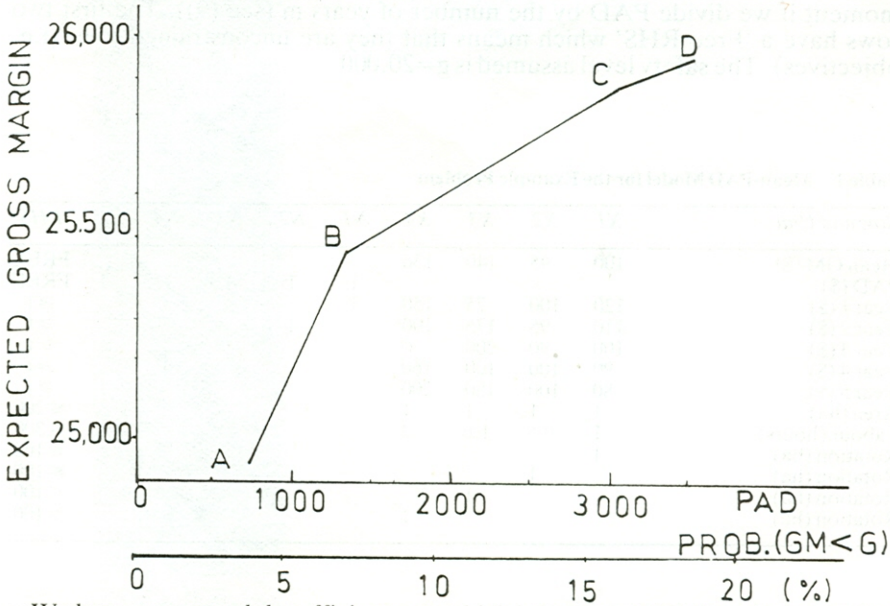
A parametric analysis of t can be carried out as in Watts *et al.*, by generating an efficient set for each t. This analysis shows that the different sets have most of the points in common.

The probability of failure, estimated by (9), is computed for each extreme efficient point in our example. We show that by applying it to point A in Figure 1, the probability of failure at this point is:

$$\Pr[x<20{,}000] < 738/[5^*(23{,}800-20{,}000)] = 3.88\% \qquad (11)$$

where g=20,000; t=23,800; m=number of years (m=5) and PAD=738.

**Figure 1 Mean-PAD Efficient Set (t=23,00)**




We have generated the efficient set, which is the aim of MOP techniques, but some authors consider that this is only the first stage in the solution of a multiobjective problem (Rehman and Romero, 1985). The decision-maker may be interested in obtaining a definite solution. In that case we may tackle the second stage by Compromise Programming (Zeleny, 1982) or resorting to some interactive techniques (see Cohon, 1978).

The trade-offs between the mean gross margin and the probability of failure or the associated PAD are established. It is not very interesting to carry out a parametric analysis of g because that will not have a very intuitive meaning to the decision-maker. The value of g (disaster level) is fixed exogenously and depends on the particular conditions of the decision maker. Parameter t is fixed by the analyst in order to estimate the probability of failure.

The lower limit on the parameter t is g and the selected value of t should always generate a positive moment to avoid dominated solutions whichever is more restrictive. The upper limit on t is the value of the profit-maximising solution. Atwood (1985) proposes a method to determine t endogenously.

For an application of the mean-PAD method to a real decision-making problem for various types of family farm in the horticultural sector, see Berbel (1986, 1987). The complete model includes income, risk, leisure, and seasonal labour. The results show that this method can be a useful tool in applied farm planning.

## Conclusions

The analysis of risk should be carried out in a multiobjective context by studying the trade-offs between risk and return. Generally return is measured as the expected income and risk as some statistical moment which measures variability. Some authors prefer to measure risk as probability of failure or any measure of 'downside risk'.

In a Linear Programming context, the use of a mean-PAD model is proposed as a better alternative to the MOTAD models. The use of inequalities for lower partial moments can generate upper bounds to safety-first type probabilities. This allows the substitution of variance or MOTAD parameters for a more intuitive estimation of failure. The use of concepts such as probability of failure of not achieving a desired target return will make the presentation of results clearer thus improving the use of interactive techniques. An intuitive meaning to decision-makers can make easier the use of interactive techniques.

If the analysis of risk-return conflicts is done on a multiple criteria basis, the most suitable method for generating the risk-return trade-off curve can be implemented and the best compromise solution obtained. The analysis of other objectives such as leisure, environmental effects, etc. which may conflict with safety and income should be introduced in any analysis of farm planning problems.

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## APPENDIX

## DERIVATION OF ATWOOD'S INEQUALITY

The definition of the partial moment of order k respect to t is:

$$\Theta(k,t) = \left[ \int_{-\infty}^{t} (t-x)^k f(x)\, dx \right] \quad (A-1)$$

We want to estimate the probability of the variable x with the distribution function f(x) falling below a threshold level g:

$$\Theta(k,t) = \left[ \int_{-\infty}^{g} (t-x)^k f(x)\, dx \right] + \left[ \int_{g}^{t} (t-x)^k f(x)\, dx \right] \quad (A-2)$$

The second integral of (A−2) is always positive for k≥1 because the function (t−x) cannot be negative. Thus we have:

$$\Theta(k,t) \geqslant \left[ \int_{-\infty}^{g} (t-x)^k f(x)\, dx \right] \quad (A-3)$$

The greater value of (t−k) in the interval (−∞,g) is (t−g), so:

$$\Theta(k,t) \geqslant (t-g)^k \left[ \int_{-\infty}^{g} f(x)\, dx \right] = (t-g)^k \Pr(x<g) \quad (A-4)$$

The integral is the probability of x falling below g. From (A−4), we get:

$$\Pr(x<g) \leqslant \Theta(k,t)/(t-g)^k \quad (A-5)$$

If we use the first moment with respect to t [0(1,t)], and we call it Partial Absolute Deviation, we obtain:

$$\Pr(x<g) \leqslant PAD/(t-g) \quad (A-6)$$